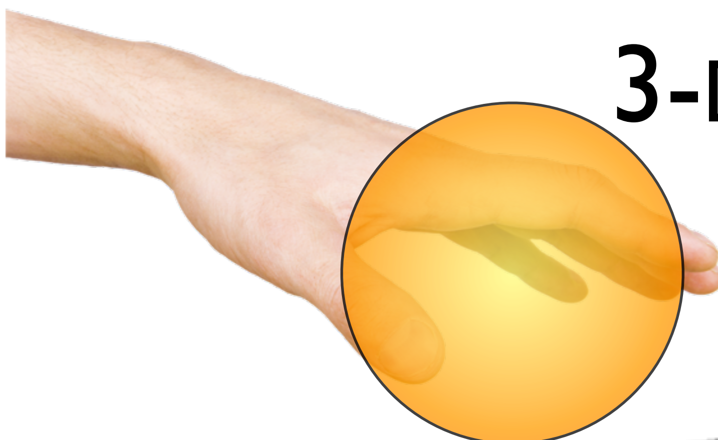
CS277 - Experimental Haptics
Lecture 13

Six-DOF Haptic Rendering I

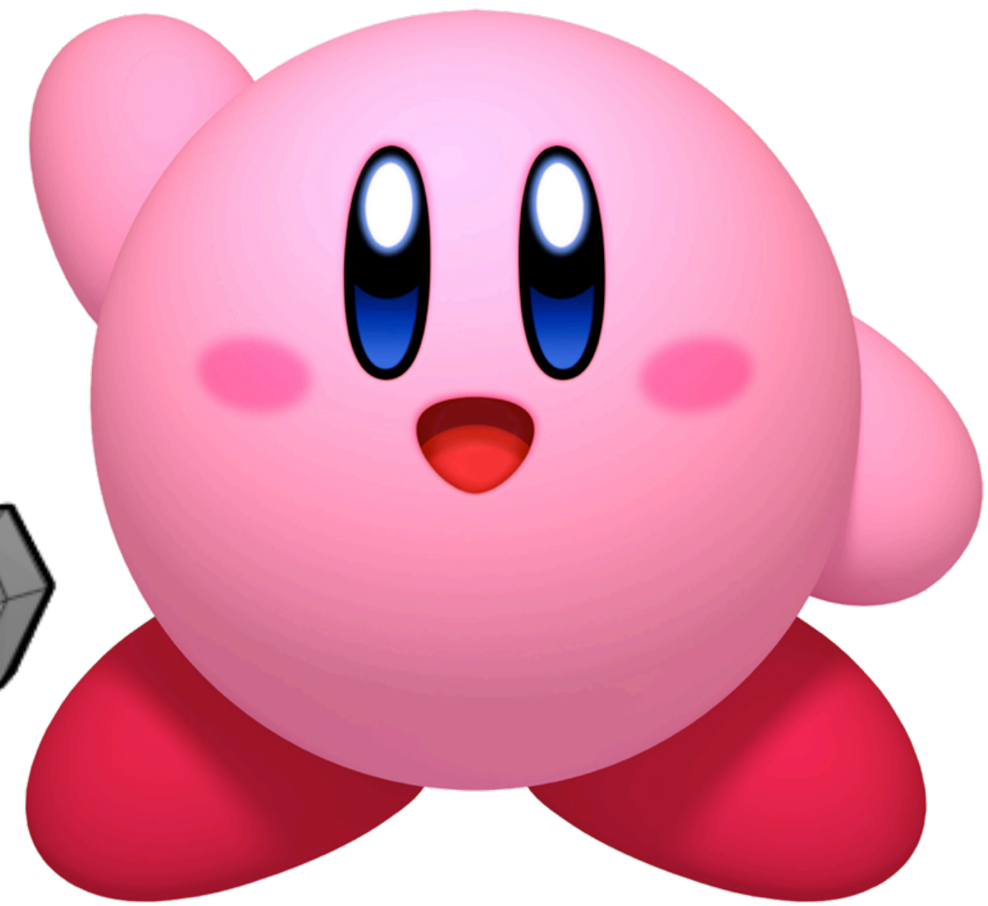
Outline

- Motivation
- Direct rendering
- Proxy-based rendering
  - Theory
  - Taxonomy

Motivation

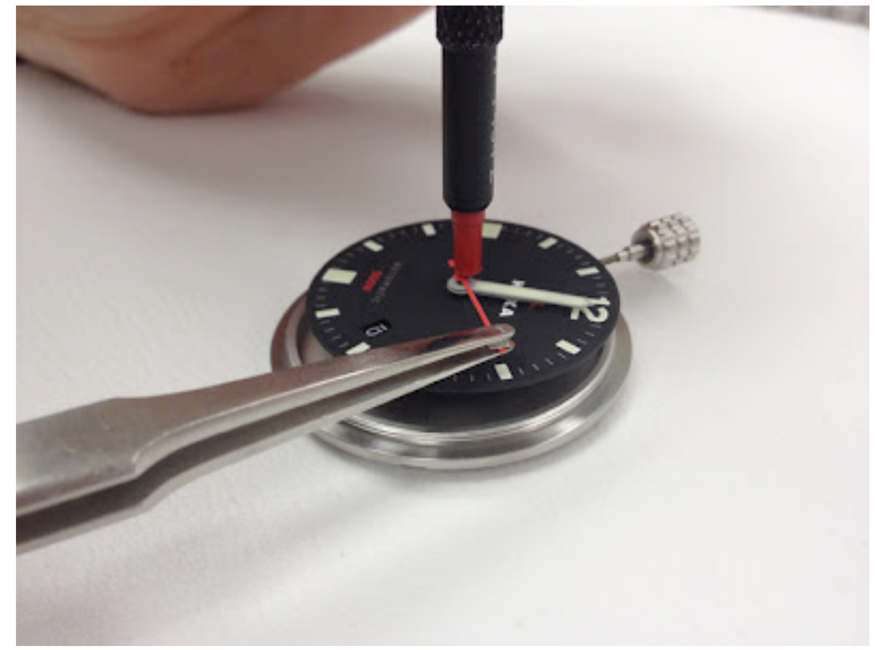


3-DOF avatar



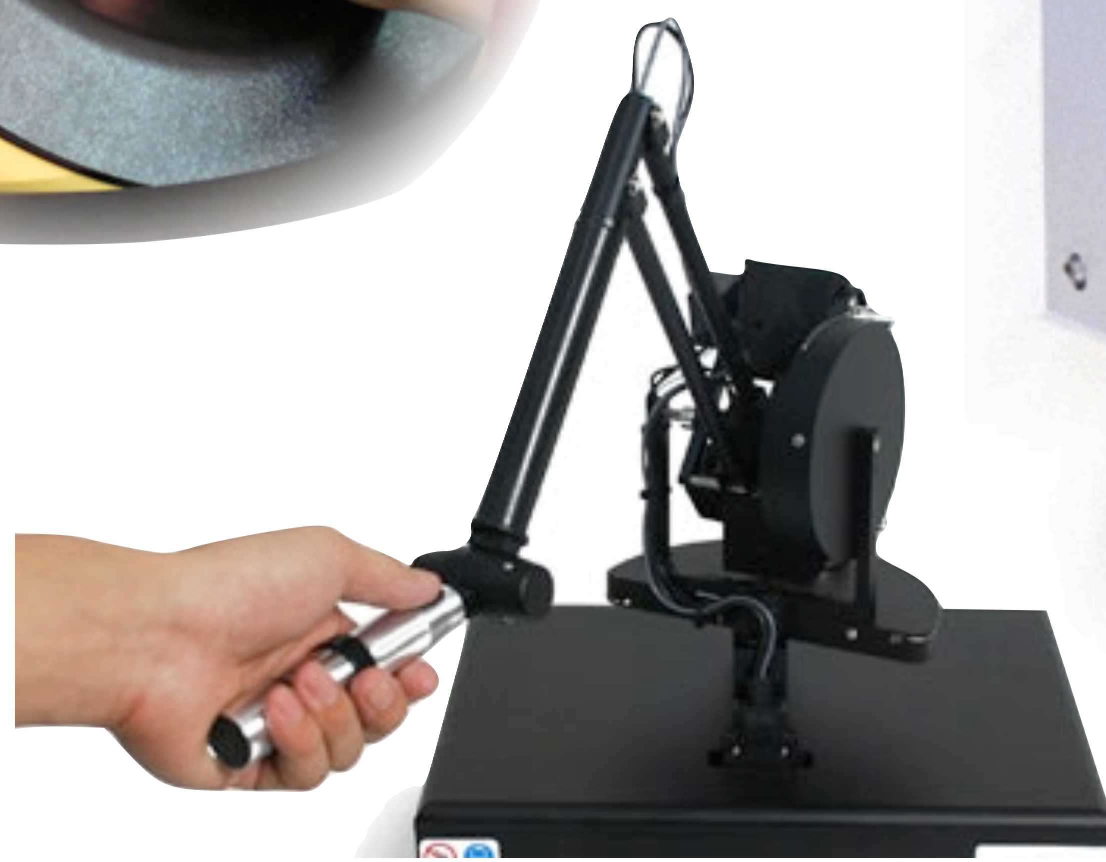
The Holy Grail?

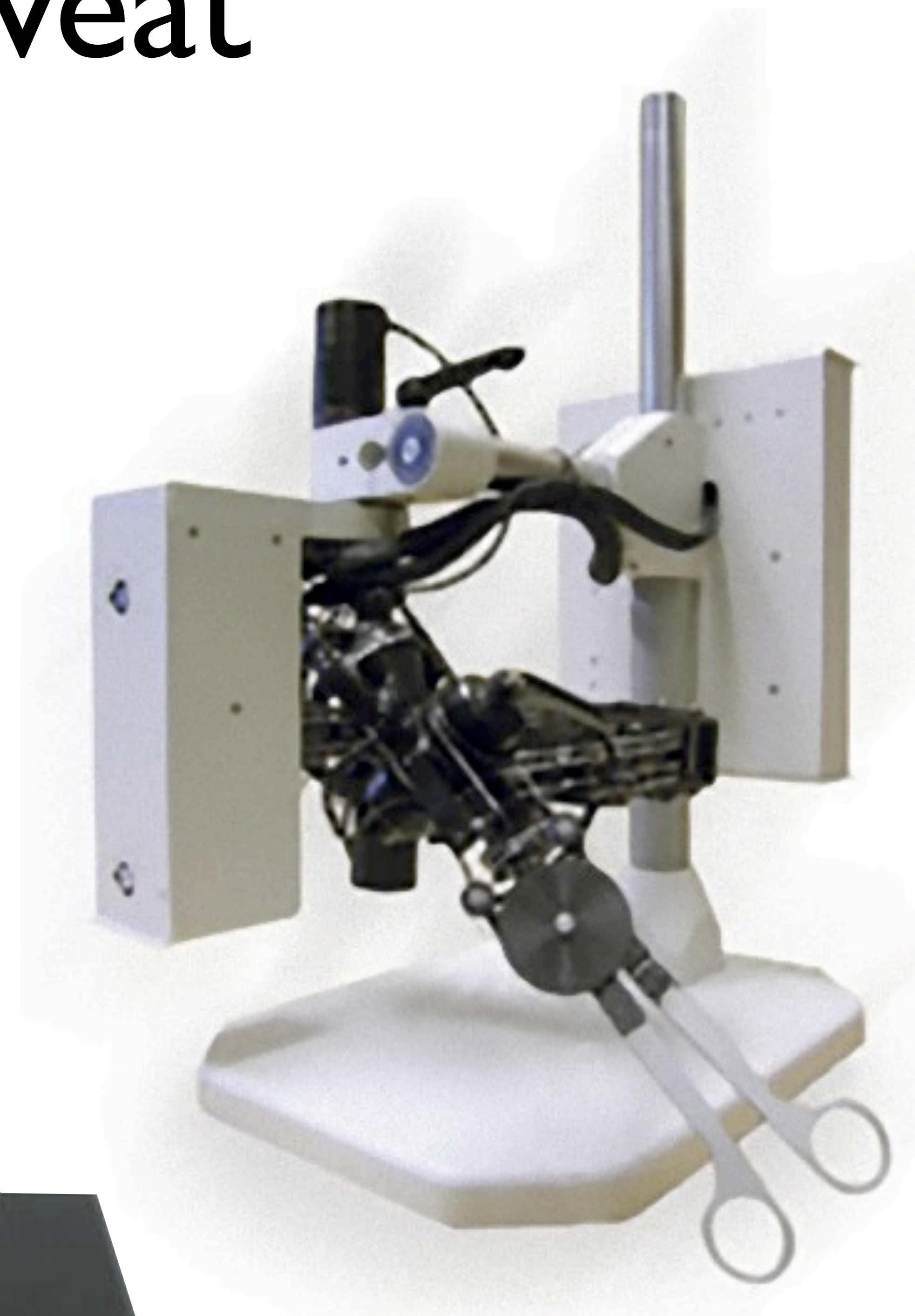
Tool-Mediated Interaction



How many degrees of freedom do we need?

One Caveat



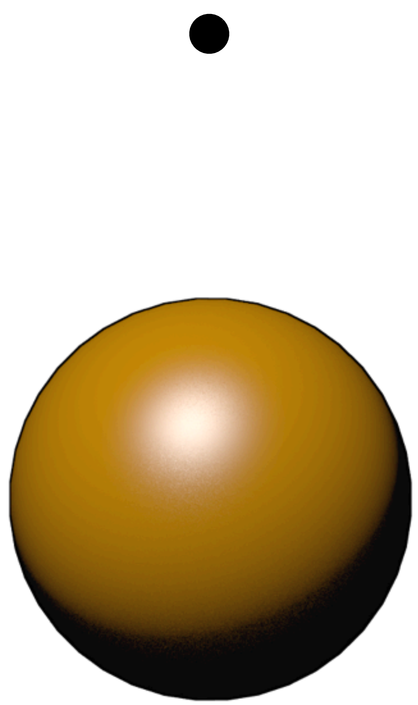


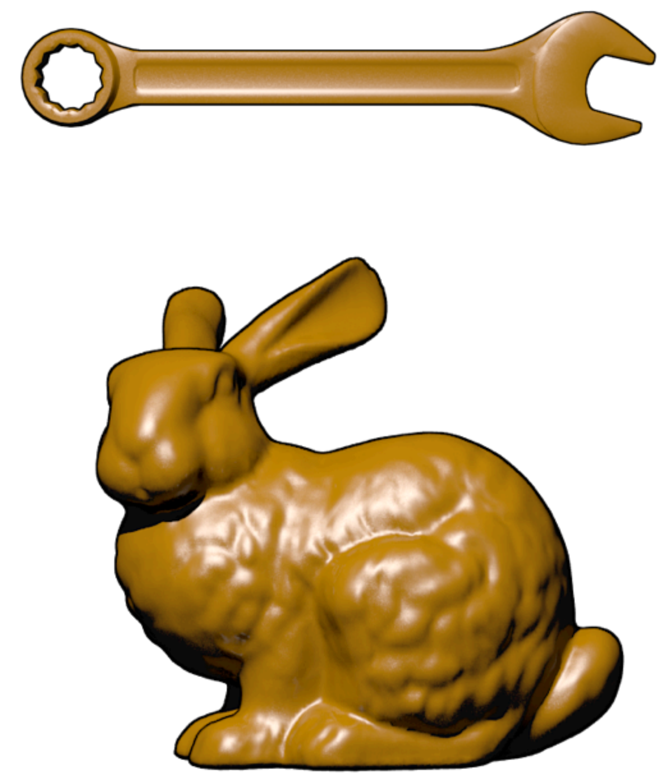
6-DOF Interaction

3-DOF
Position/Translation

6-DOF
+ Orientation/Rotation

Avatars for 6-DOF Haptics





3-DoF

Position/Translation

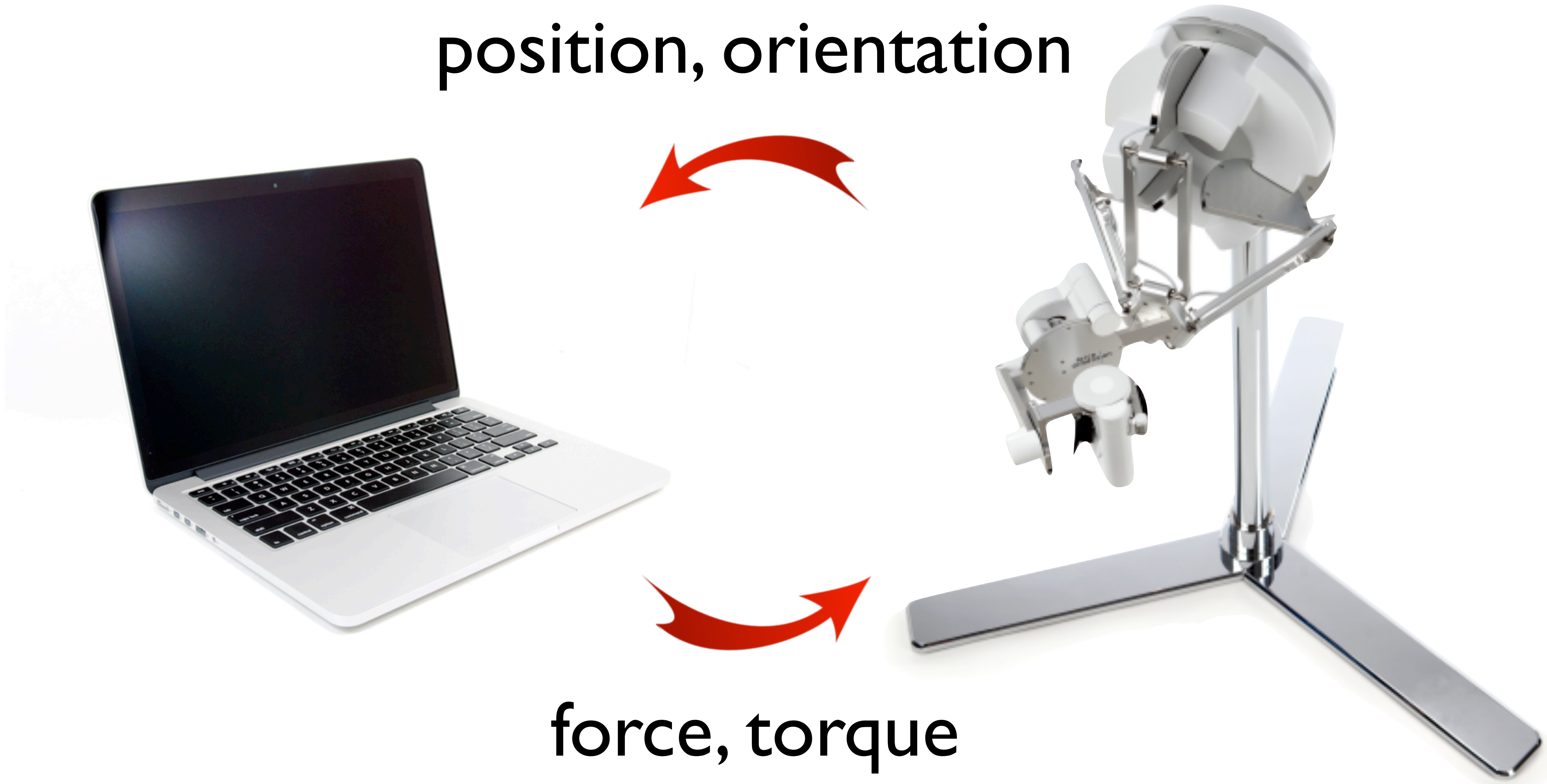
Render Force

6-DoF

+ Orientation/Rotation

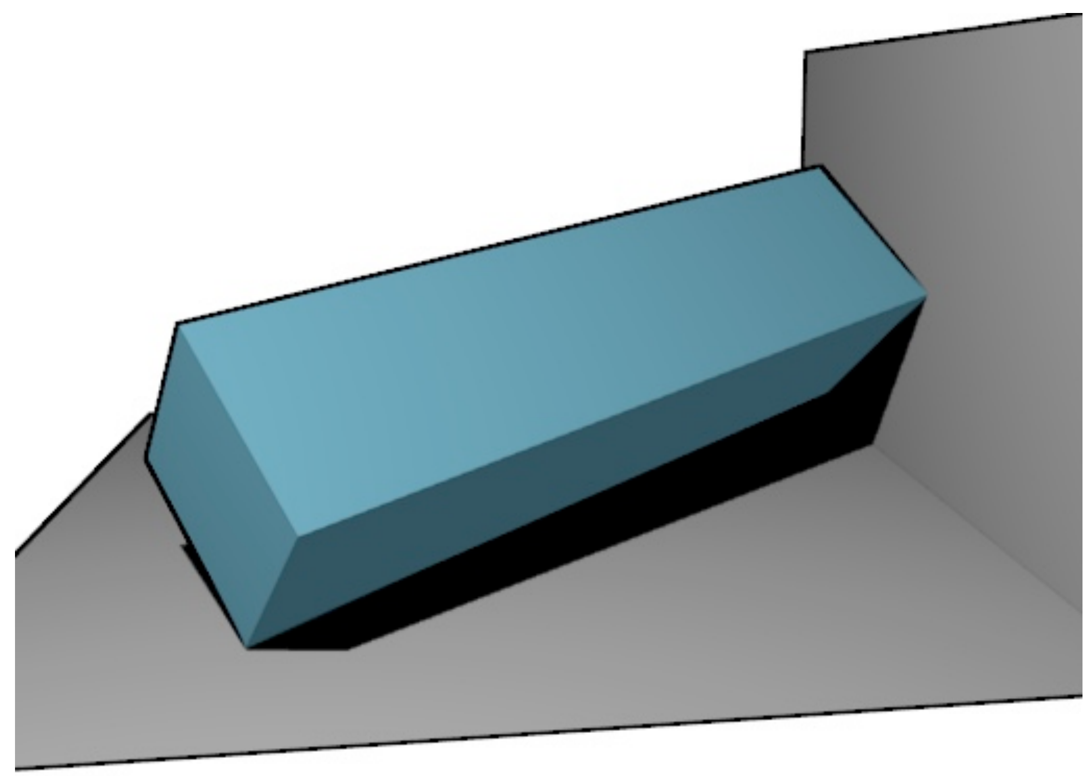
+ Render Torque

Impedance-Controlled Device

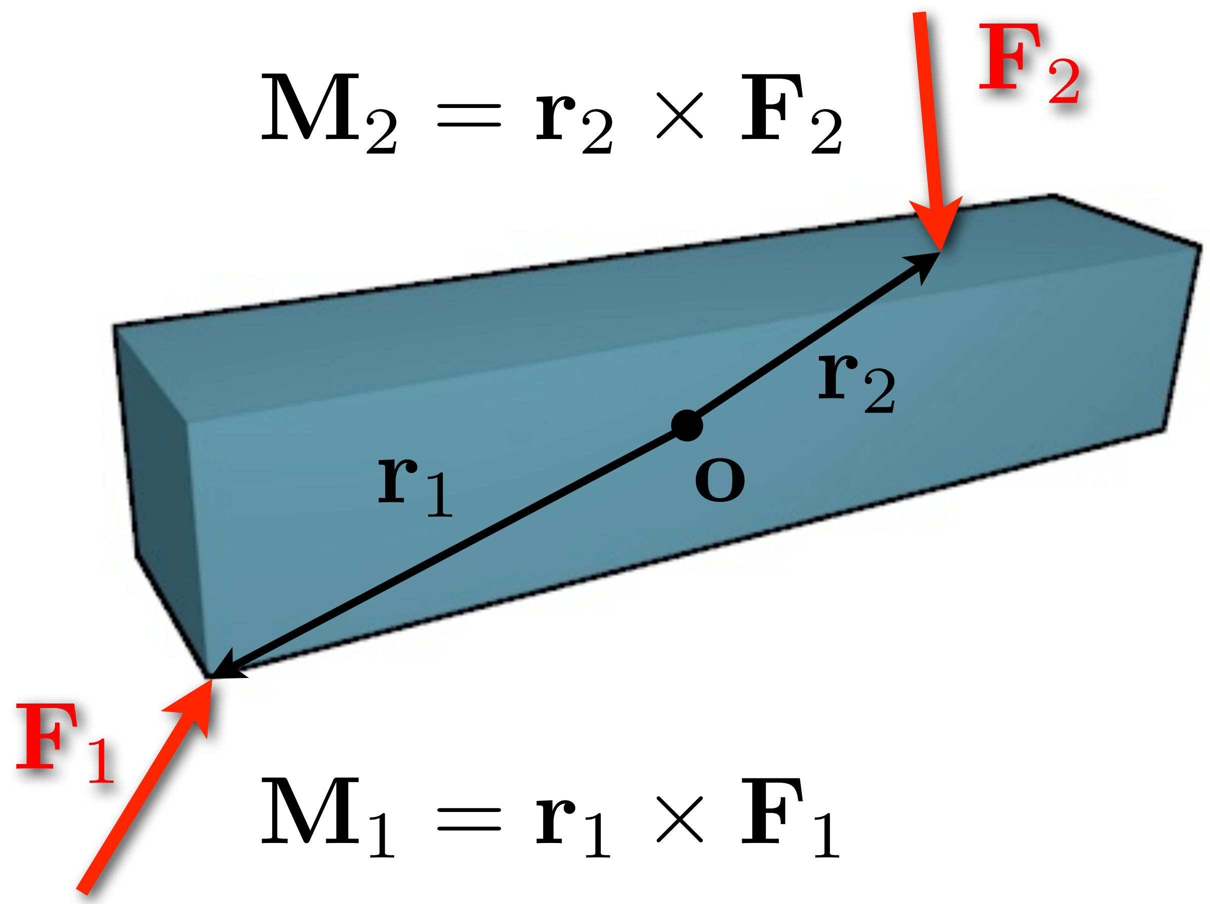


Direct Rendering

- Analogue to force field rendering
- Must consider multiple contacts in different positions for 6-DOF rendering



Forces on a Body

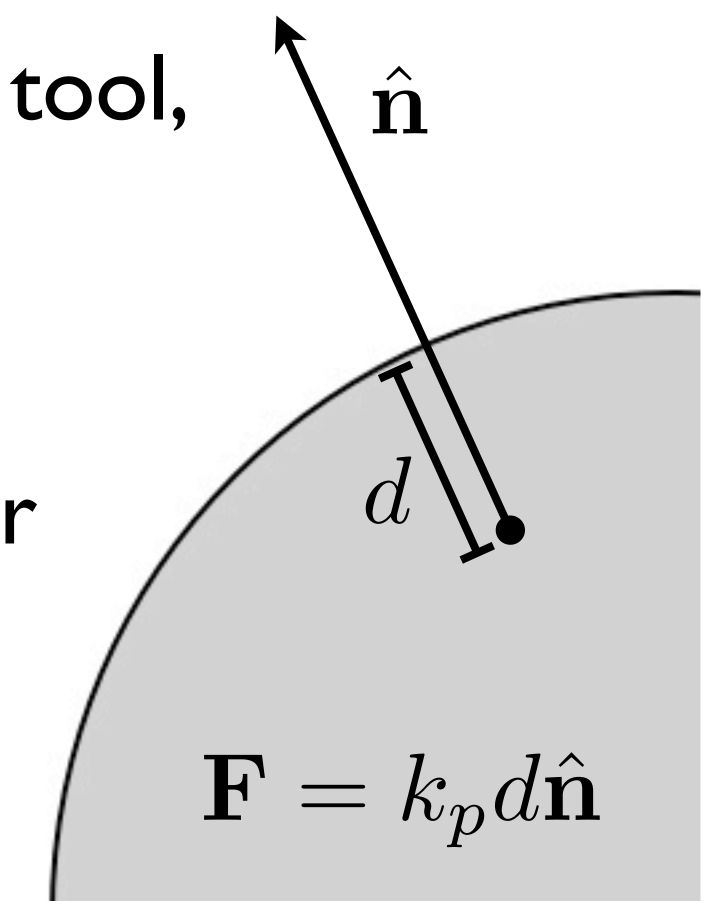


Output to Device: $\mathbf{F} = \sum_i \mathbf{F}_i \qquad \boldsymbol{\tau} = \sum_i \mathbf{M}_i$

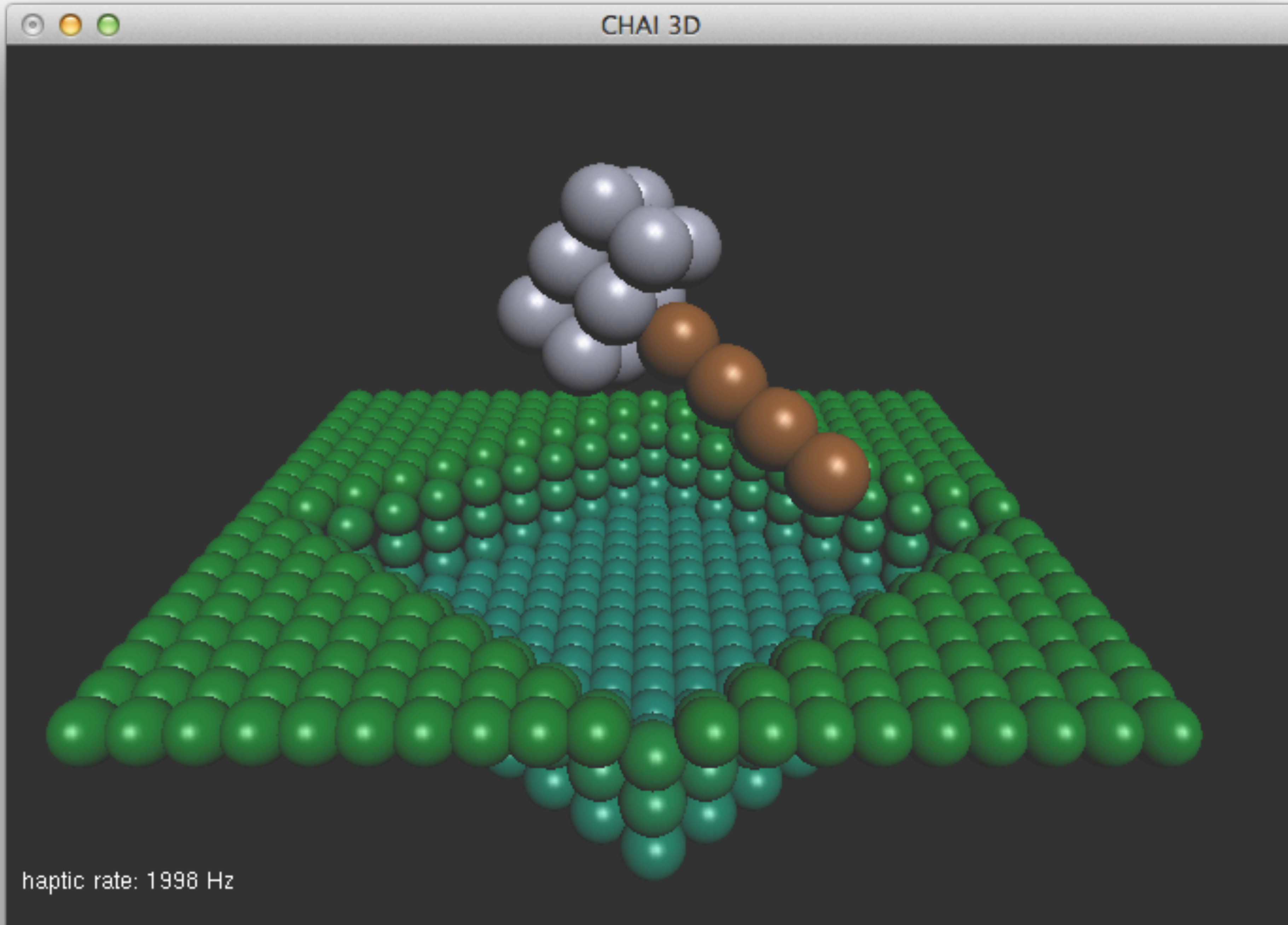
Contact Model

For each contact, you will need

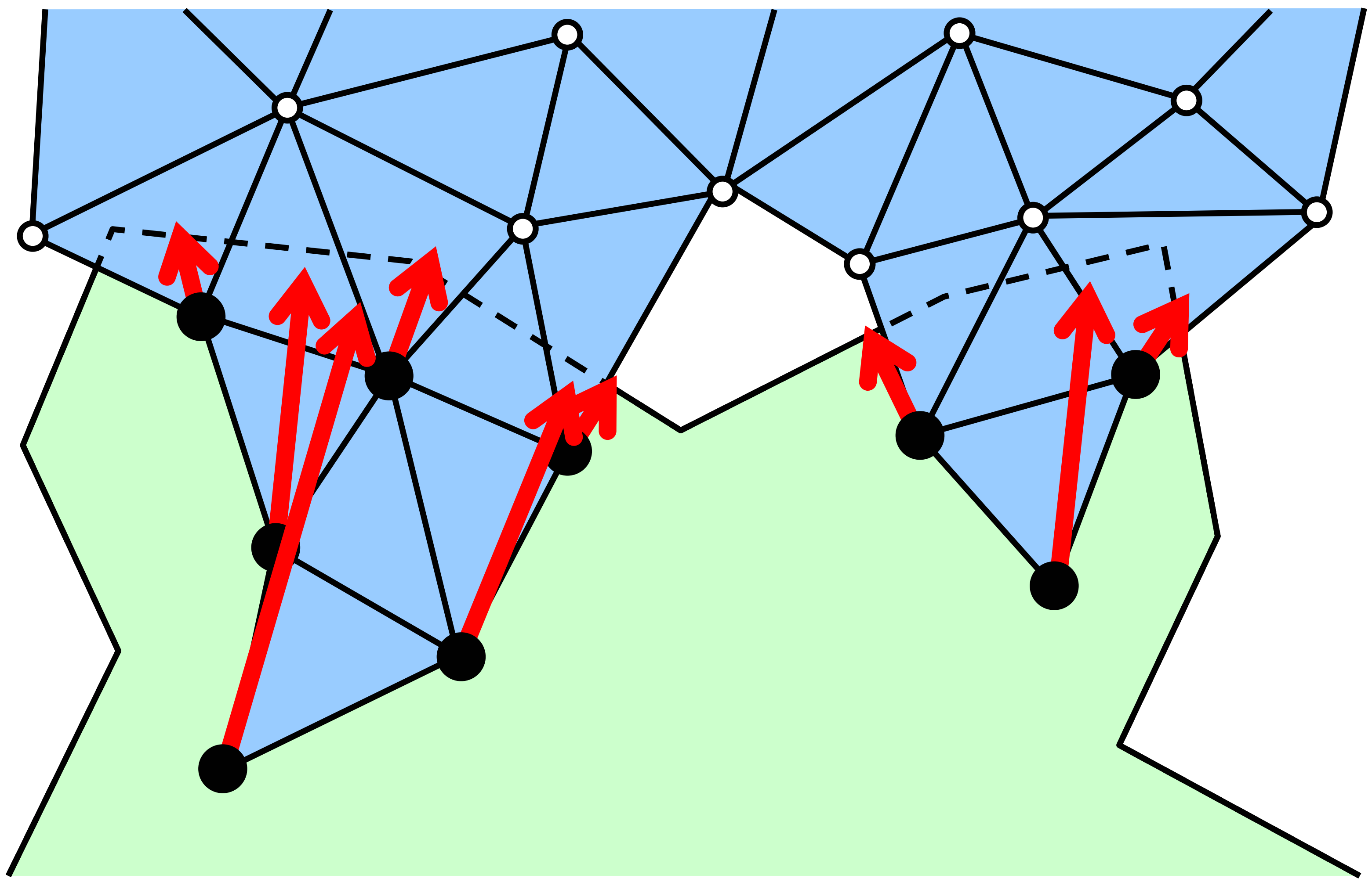
- The contact position on the tool,
- and one of
  - a force vector (magnitude + direction), or
  - a contact normal and penetration depth



Demo



Properties of Direct Rendering

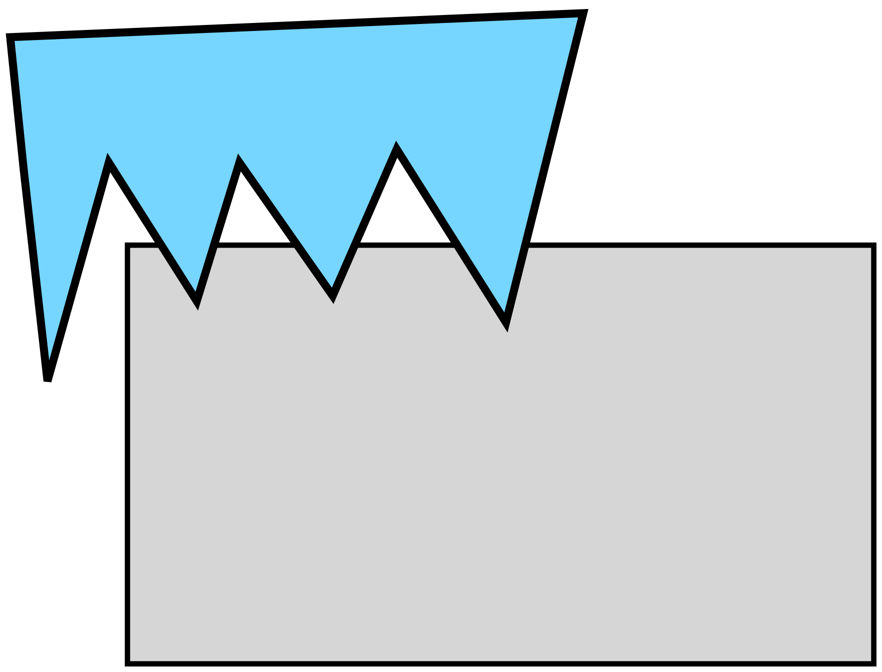


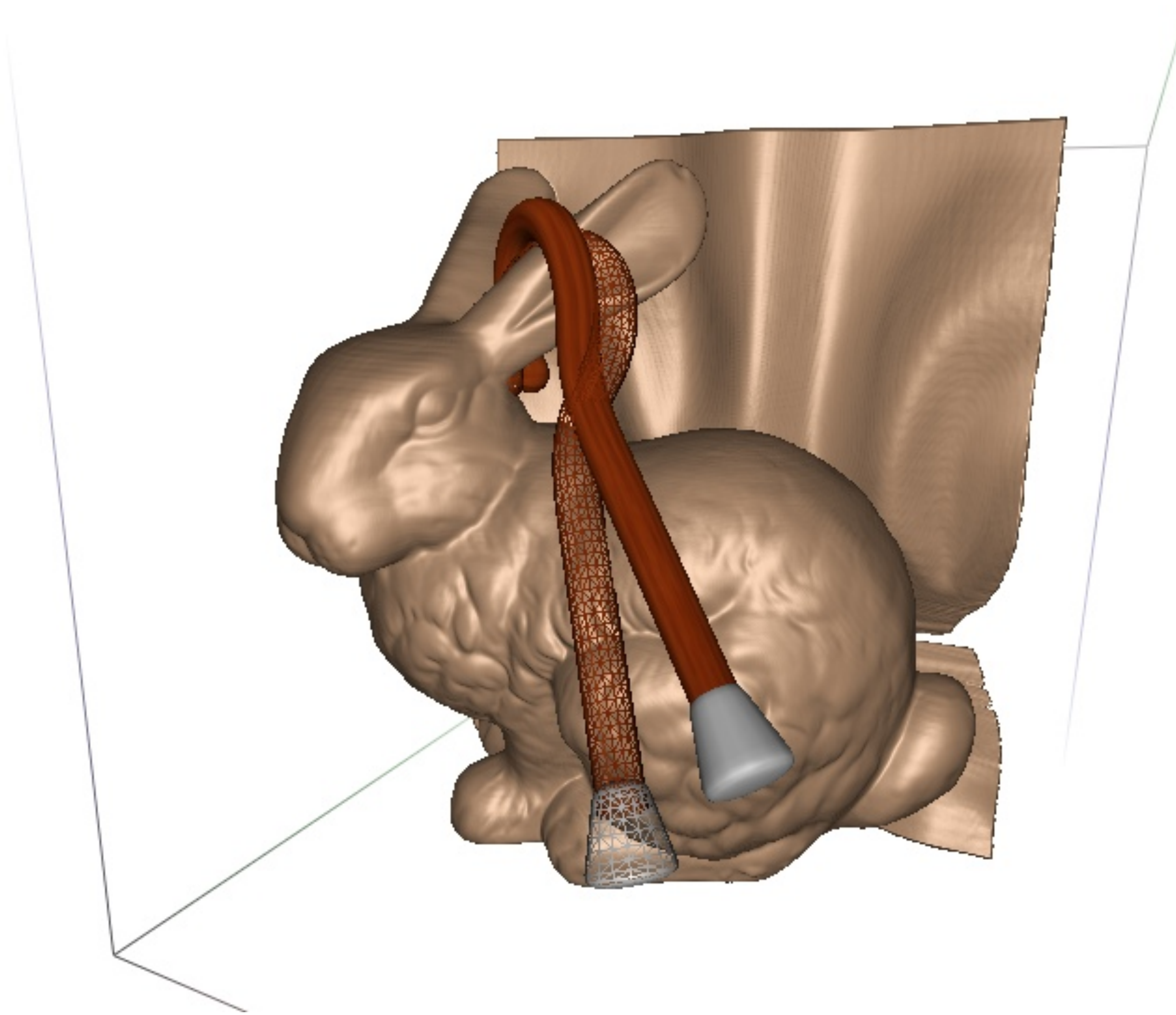
What are the advantages and disadvantages?

[From B. Heidelberger *et al.*, *Vision Modeling and Visualization*, 2004.]

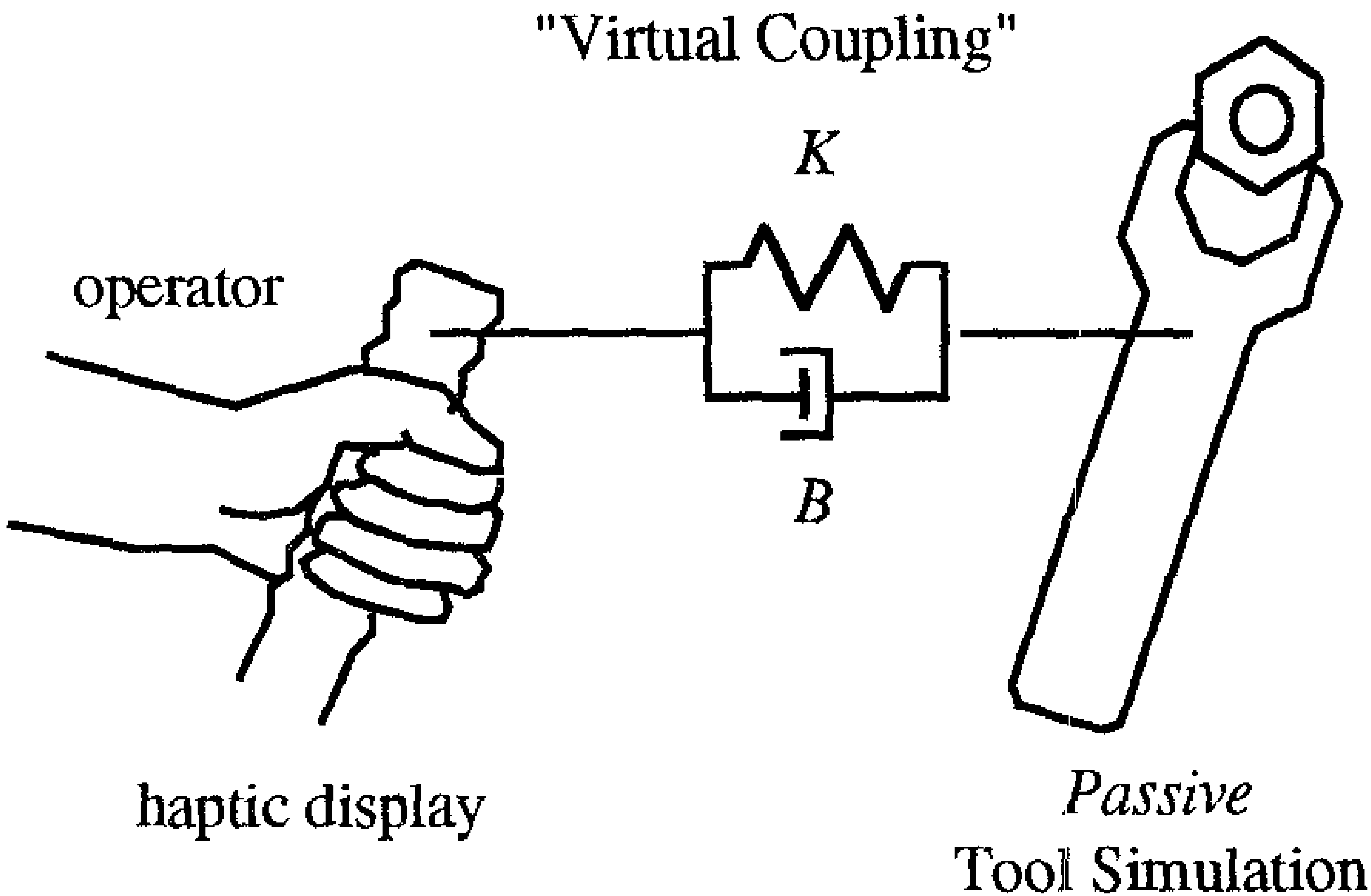
Direct Rendering Summary

- Advantages
  - Easy to implement
  - Free space feels like free space
- Limitations
  - Object interpenetration
  - Pop-through
  - Force discontinuities
  - Unbounded stiffness!





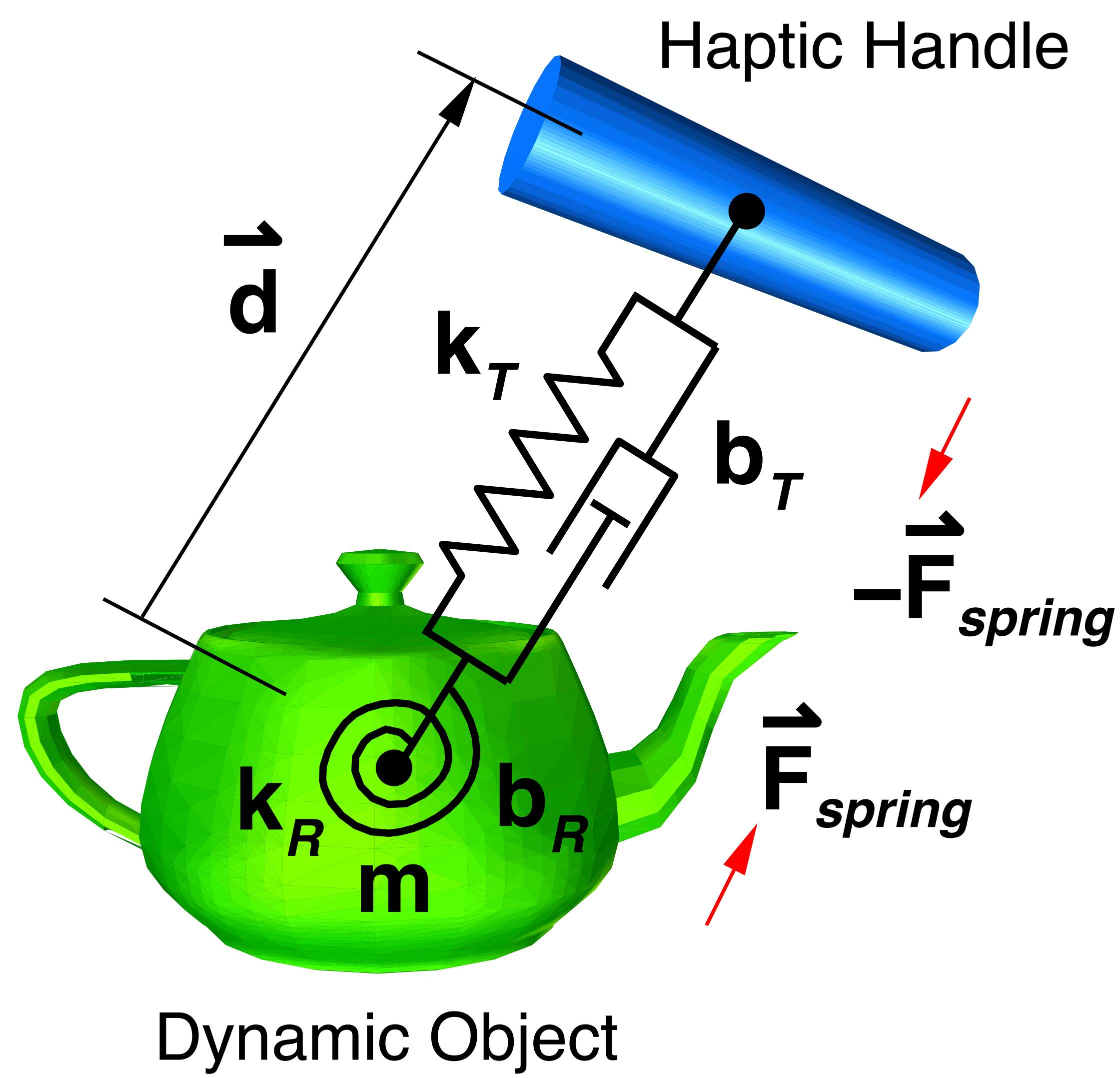
Proxy-Based Rendering



[From J. E. Colgate *et al.*, *Proc. IEEE/RSJ IROS*, 1995.]

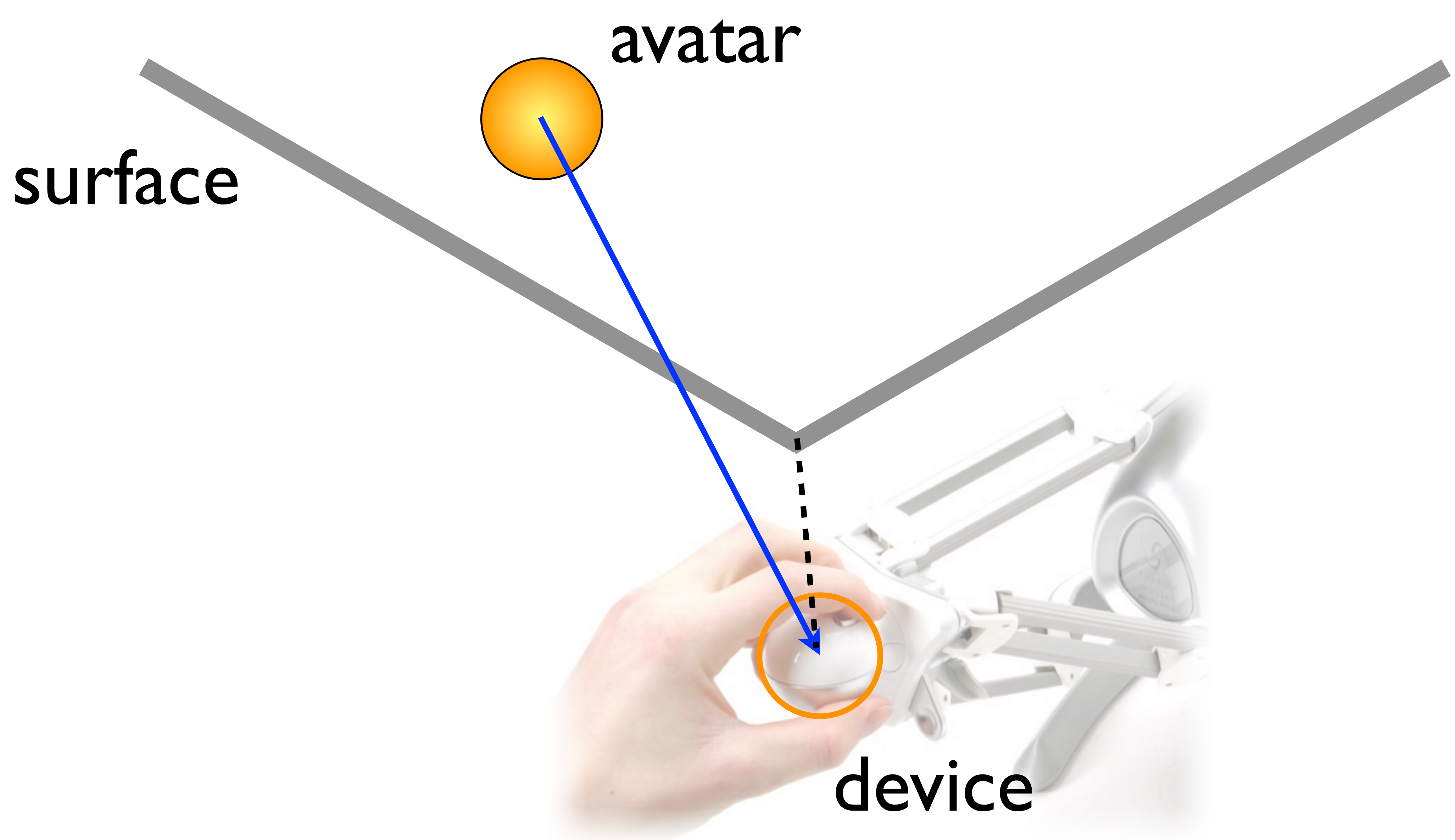
6-DOF Virtual Coupling

- Translational and rotational spring/damper coupling
  - Force proportional to displacement
  - Torque proportional to orientation difference
- Virtual walls again!

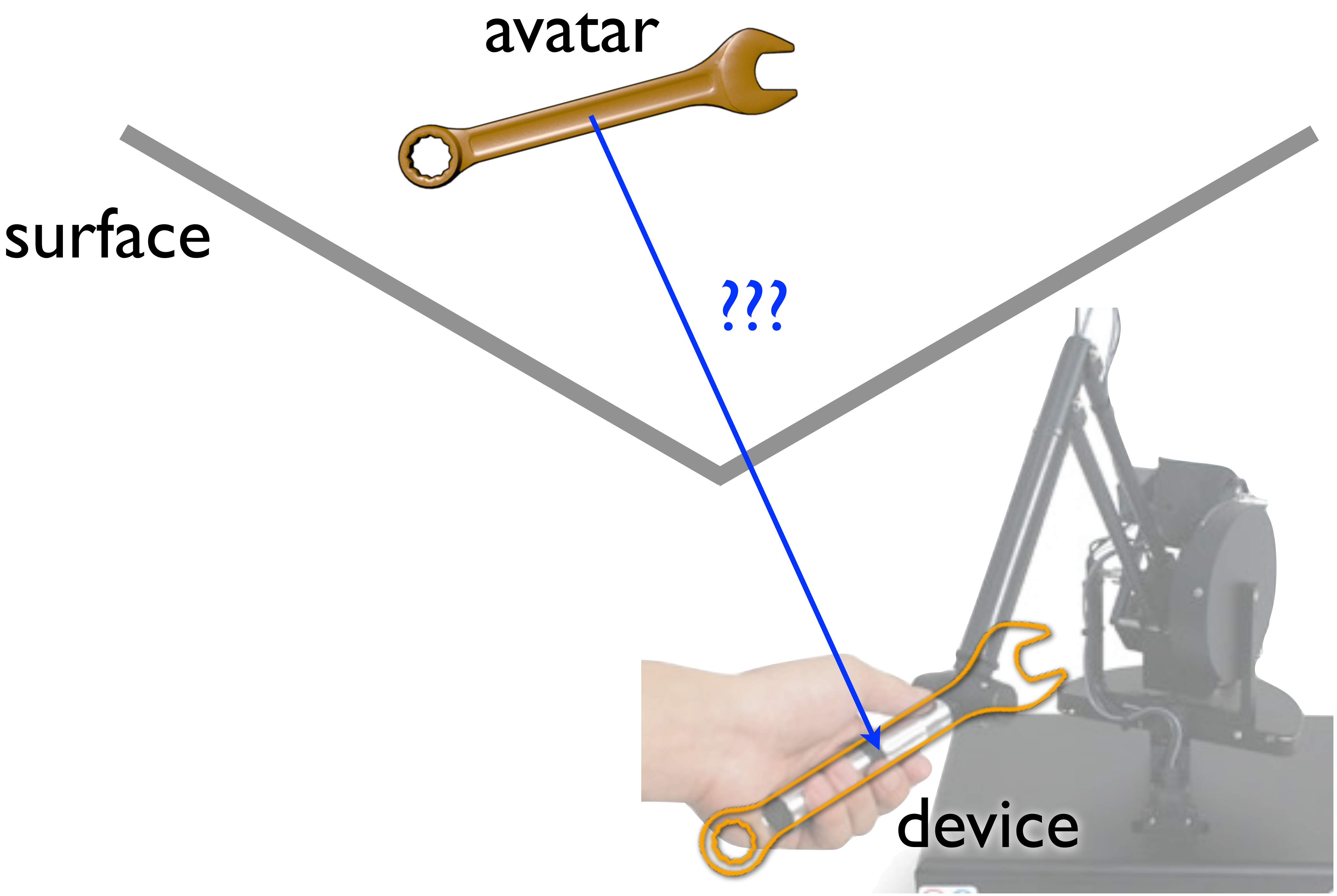


[From W. A. McNeely *et al.*, *Proc. SIGGRAPH*, 1999.]

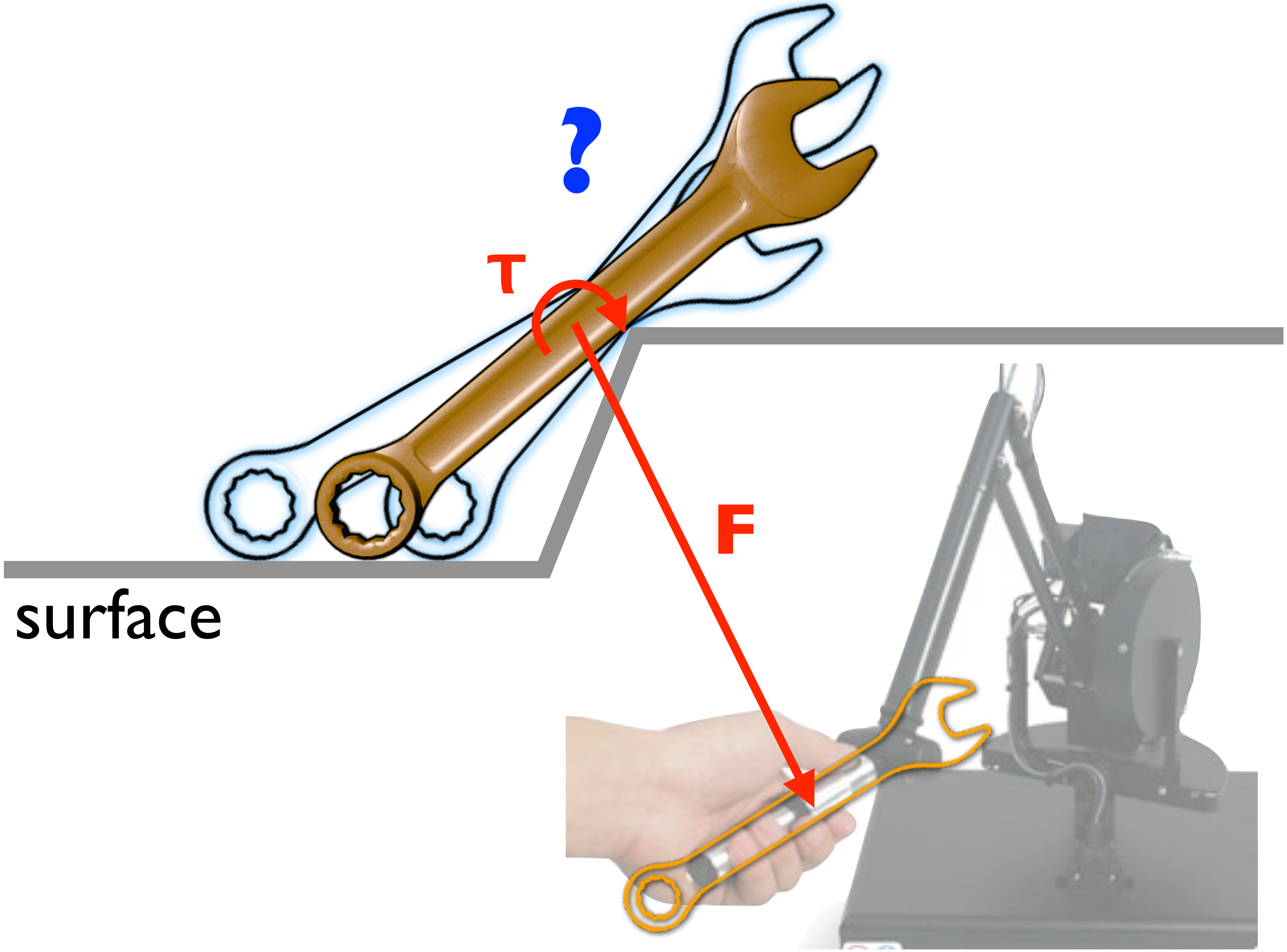
Proxy Simulation in 3-DOF



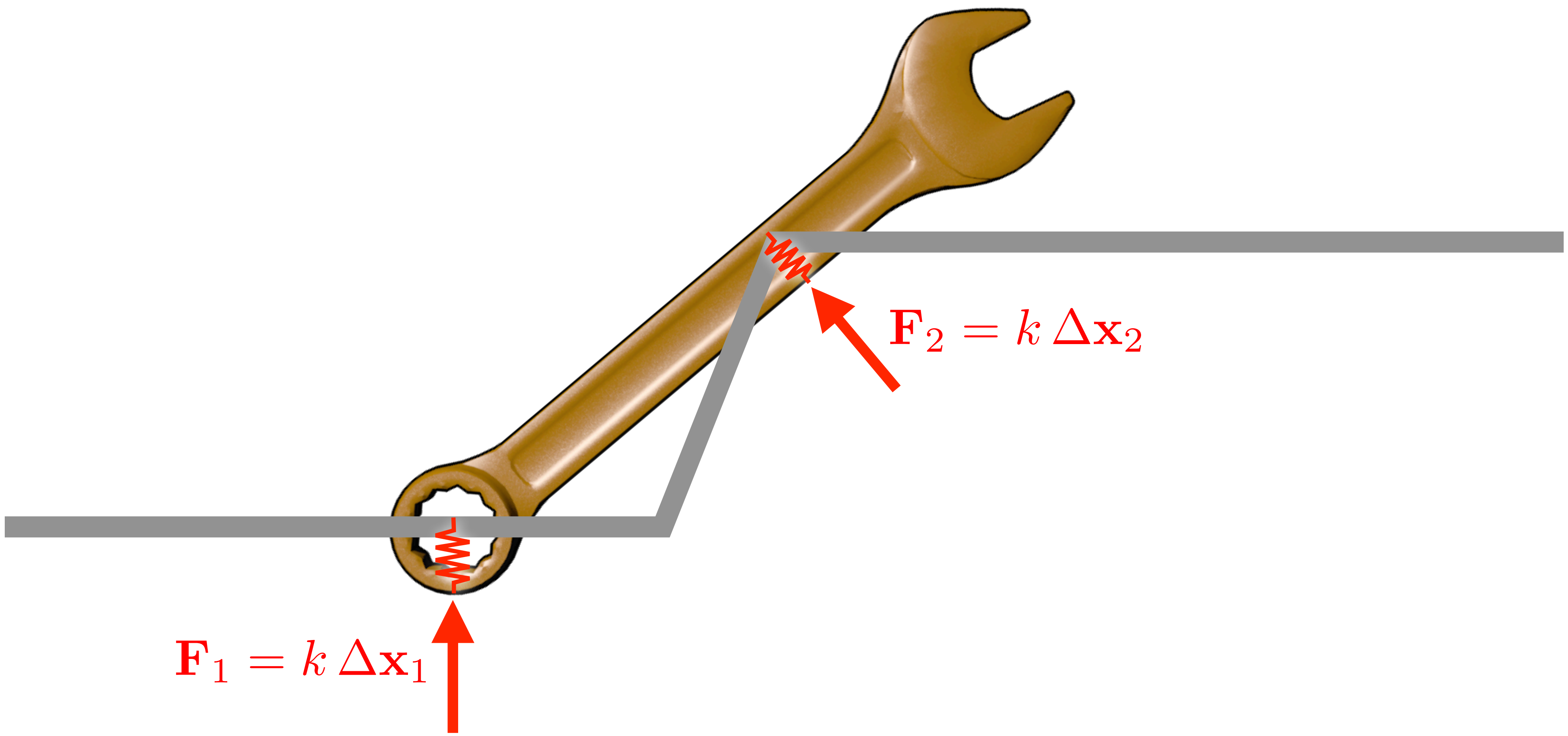
Proxy Simulation in 6-DOF



Proxy Simulation



Soft Constraints



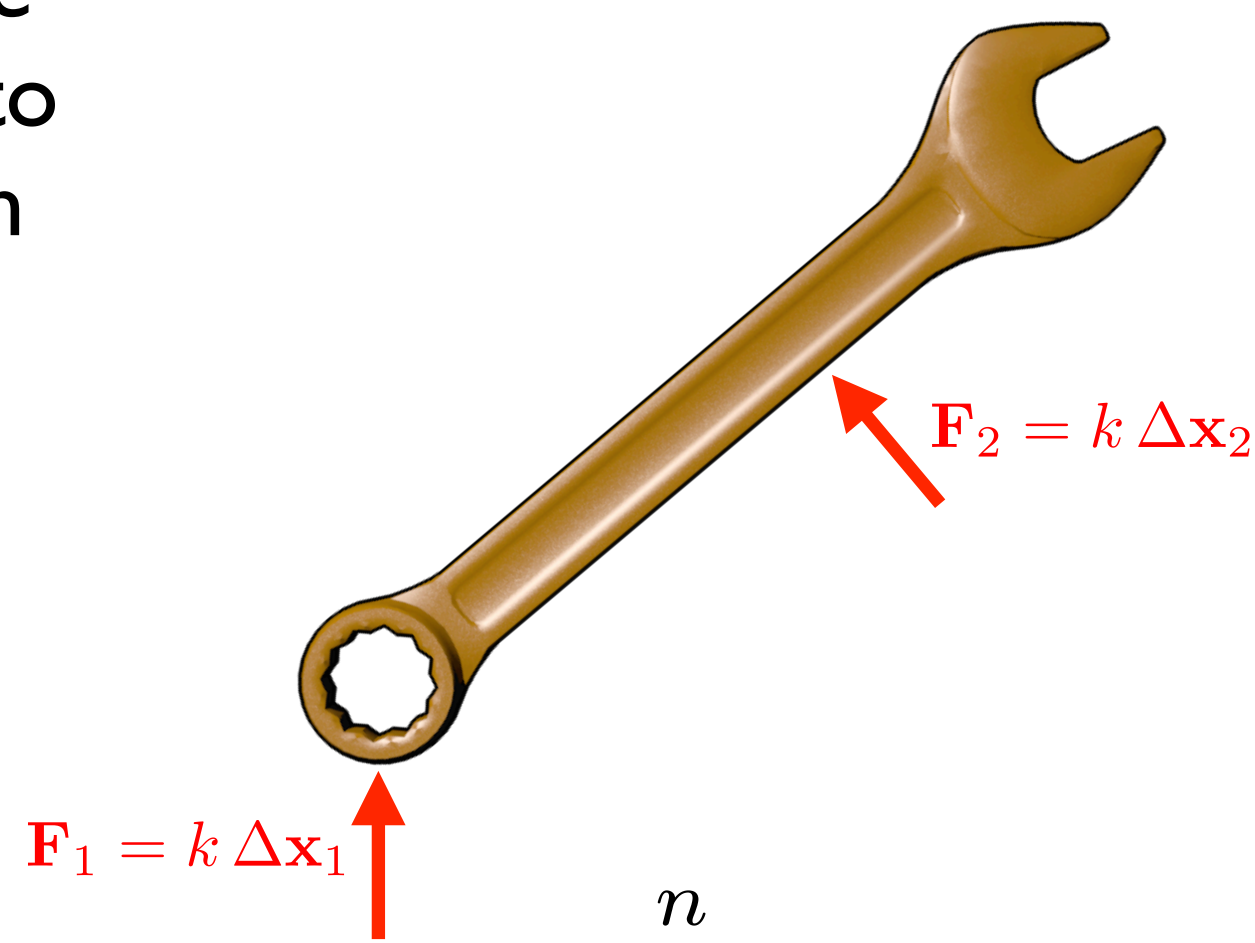
$$\mathbf{F}_{\text{net}} = \sum_{i}^{n} \mathbf{F}_i + \mathbf{F}_{\text{vc}}$$

Proxy Motion

- Numerically integrate the ODE over time to obtain **x**, the position of the avatar:

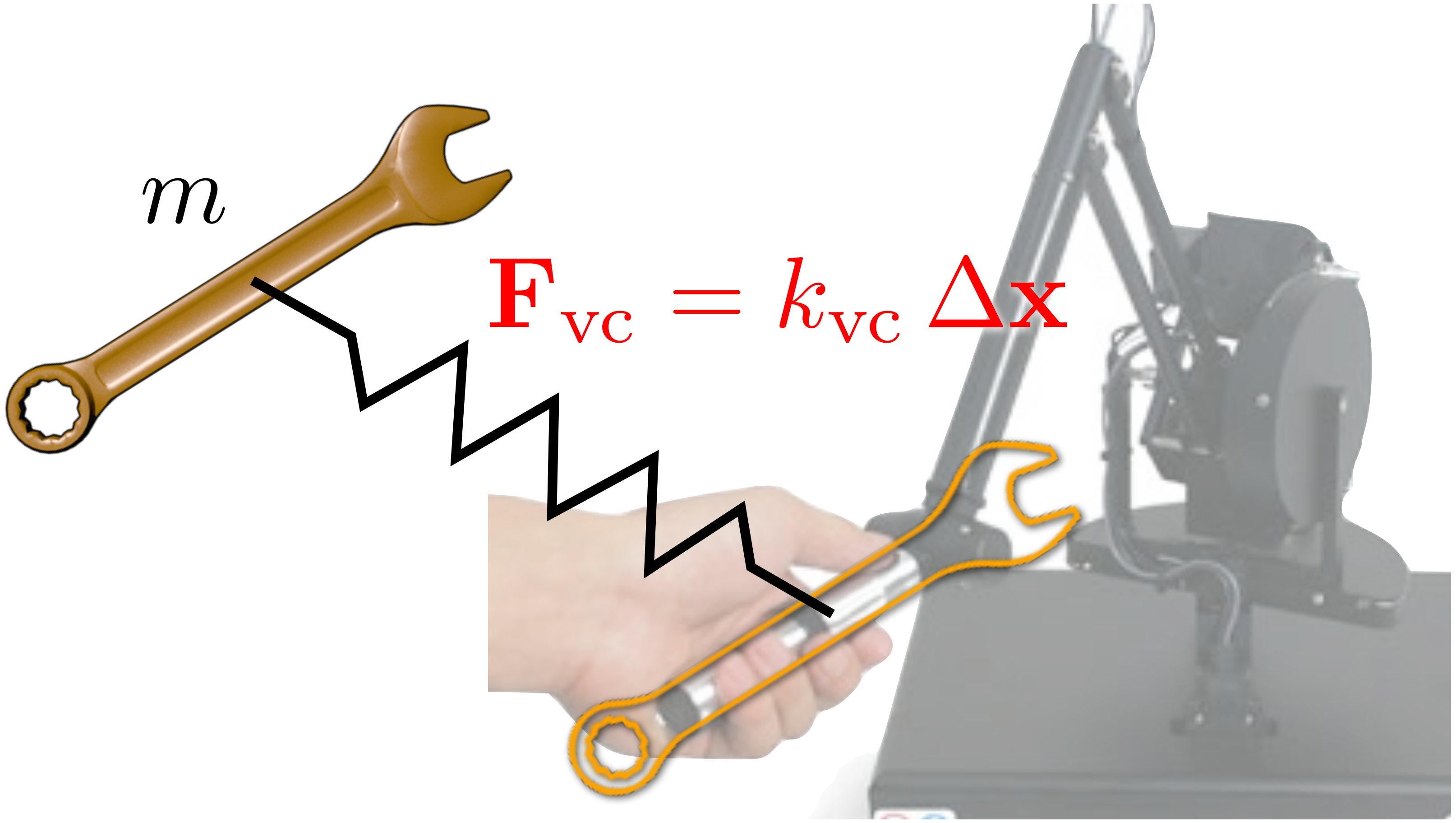
$$m\ddot{\mathbf{x}} = \mathbf{F}_{\text{net}}$$

- Do the same with moments to obtain orientation

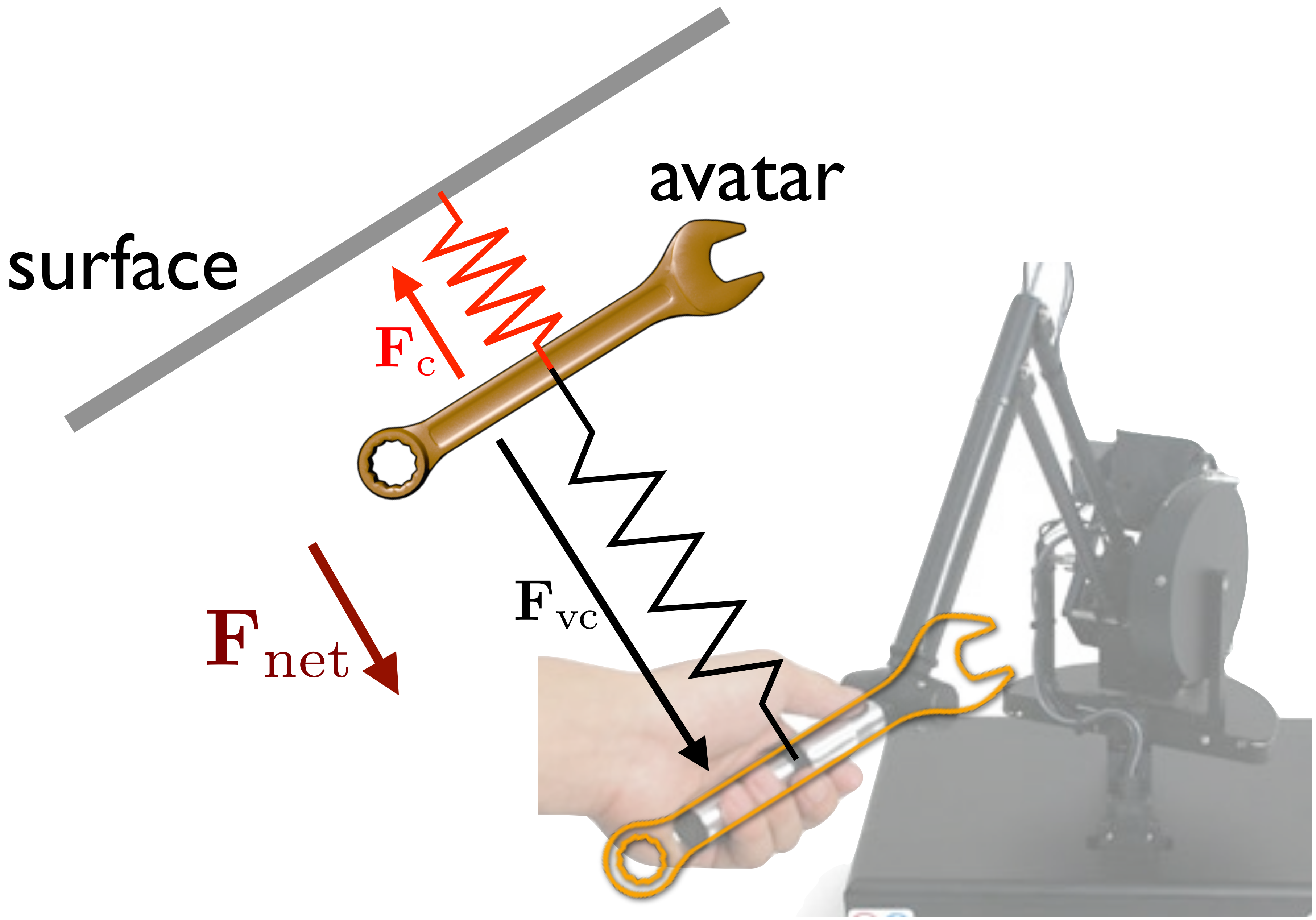


$$\mathbf{F}_{\text{net}} = \sum_i^n \mathbf{F}_i + \mathbf{F}_{\text{vc}}$$

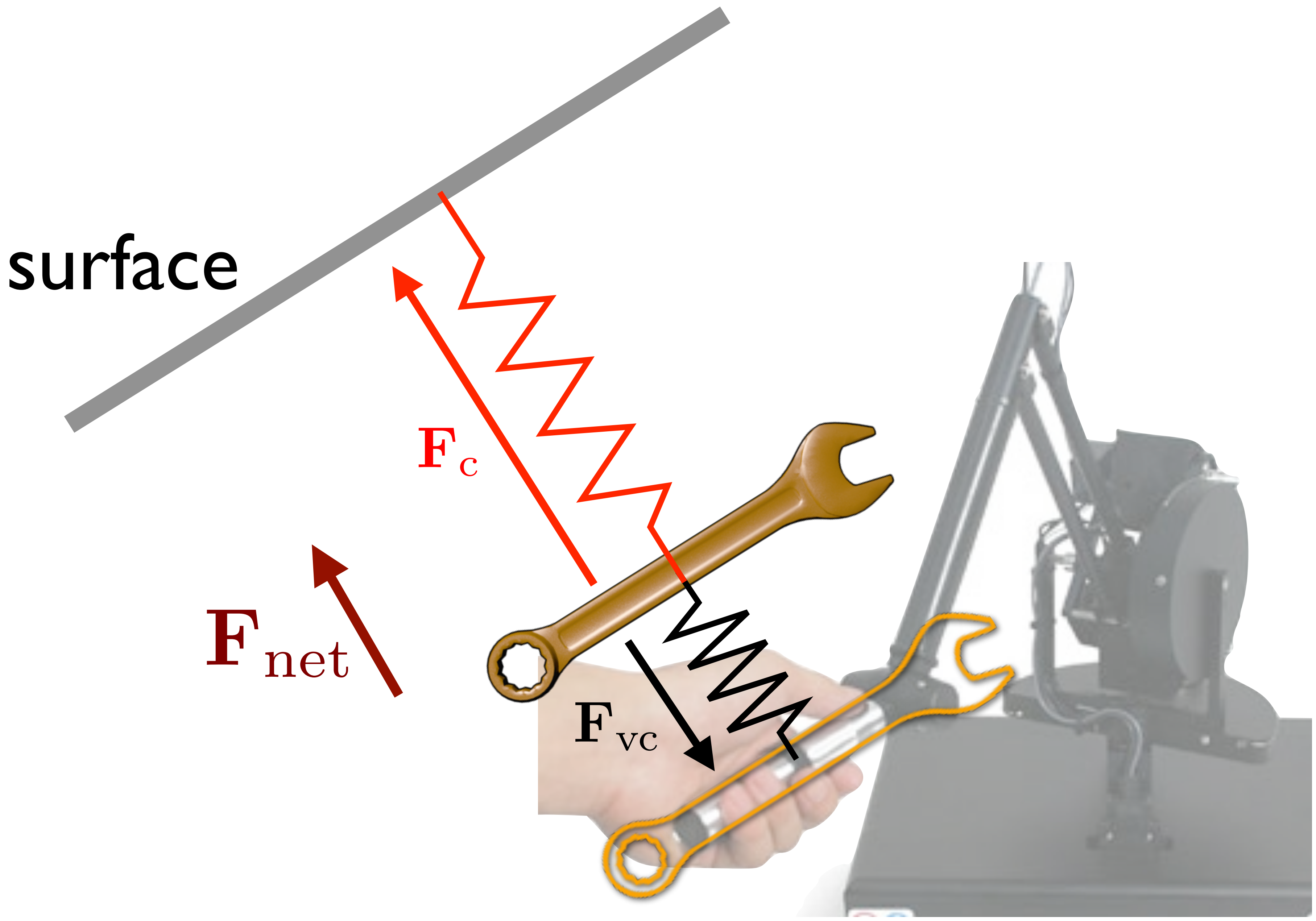
Potential Problems?



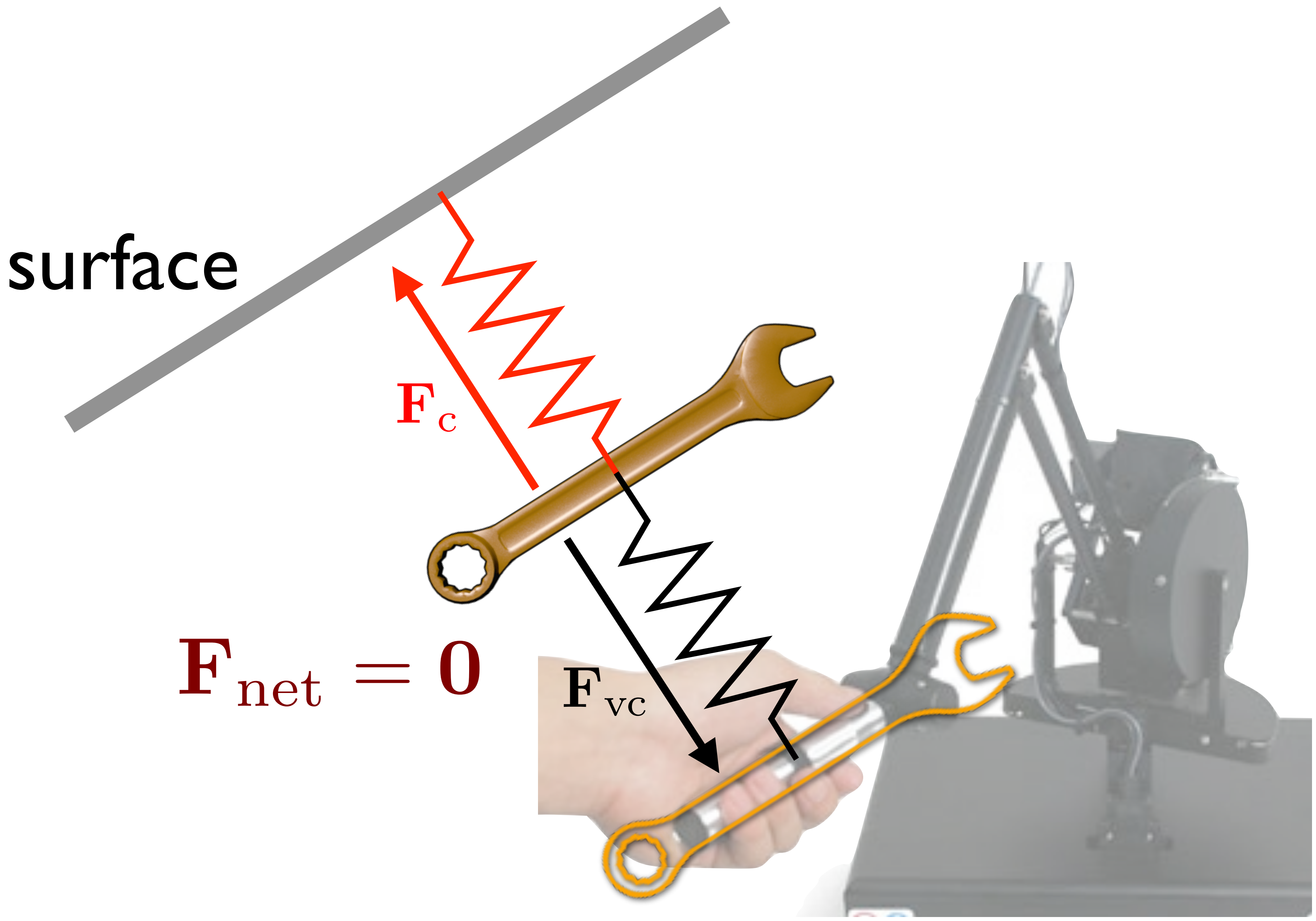
Quasi-Static Equilibrium



Quasi-Static Equilibrium



Quasi-Static Equilibrium



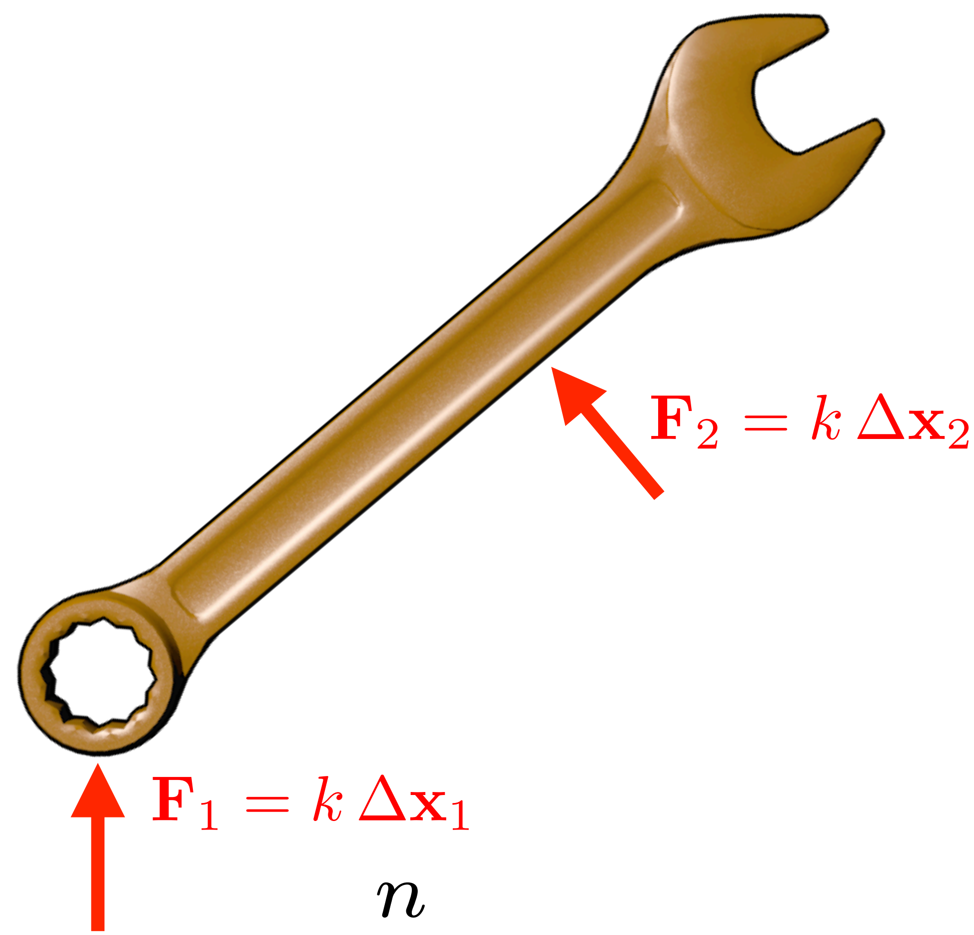
$$\mathbf{F}_{\text{net}} = \mathbf{0}$$

Quasi-Static Proxy Motion

- Solve directly for the position **x** for which the net force acting on the proxy is zero:

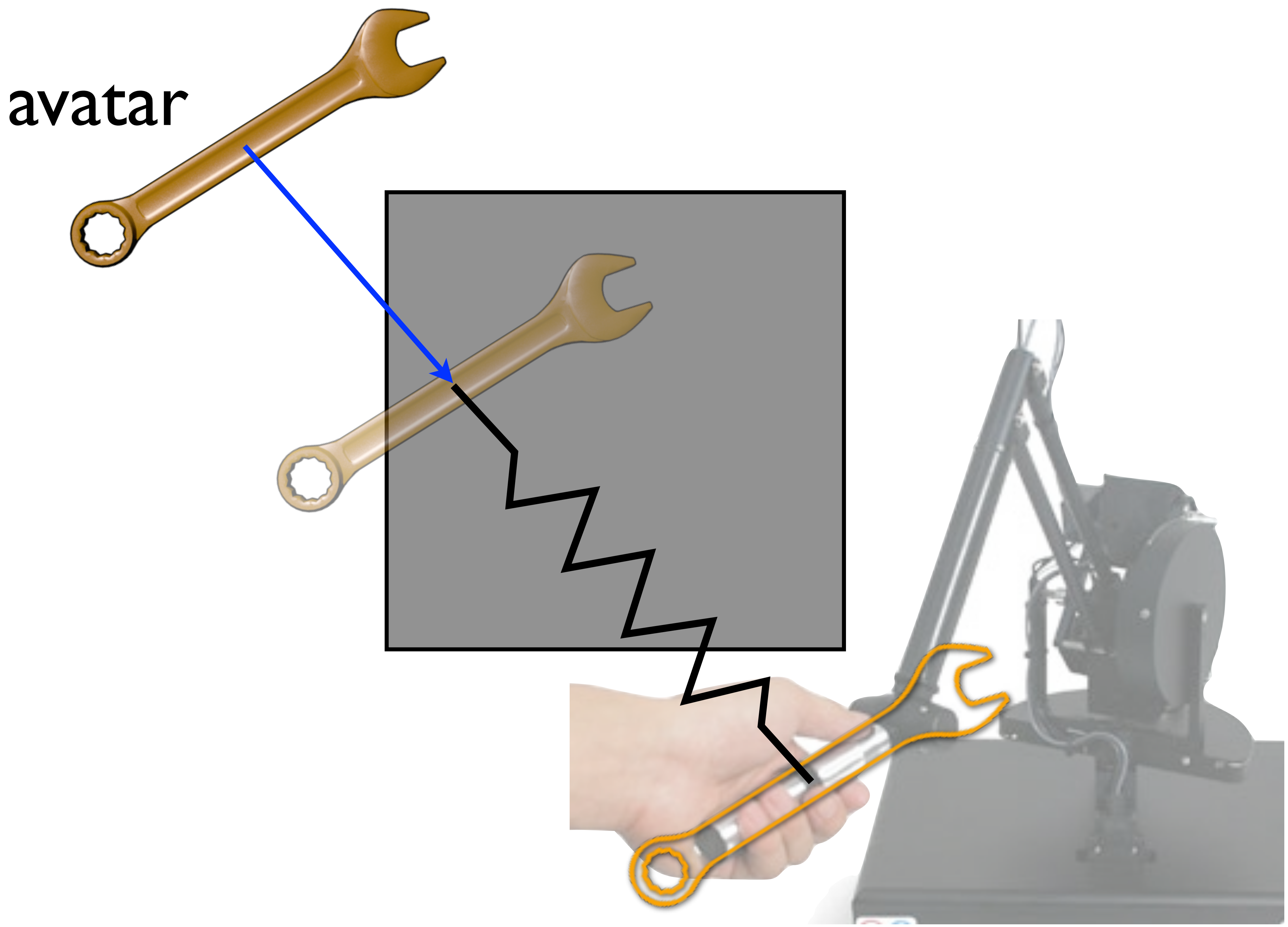
$$\sum_{i}^{n} k\,\Delta\mathbf{x}_i + k_{\mathrm{vc}}\,\Delta\mathbf{x}_{\mathrm{vc}} = \mathbf{0}$$

- Do the same with orientation to obtain net moment of zero

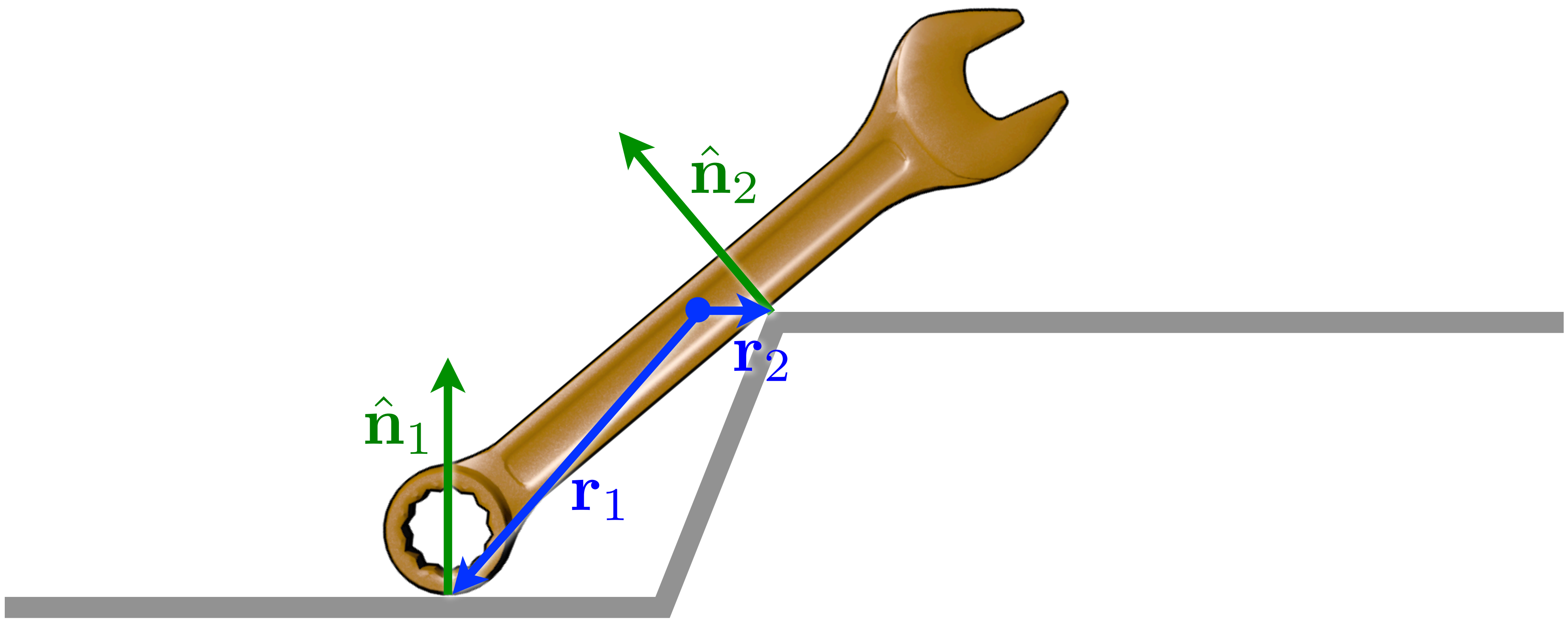


$$\mathbf{F}_{\mathrm{net}} = \sum_{i}^{n} \mathbf{F}_i + \mathbf{F}_{\mathrm{vc}}$$

Still Problems?



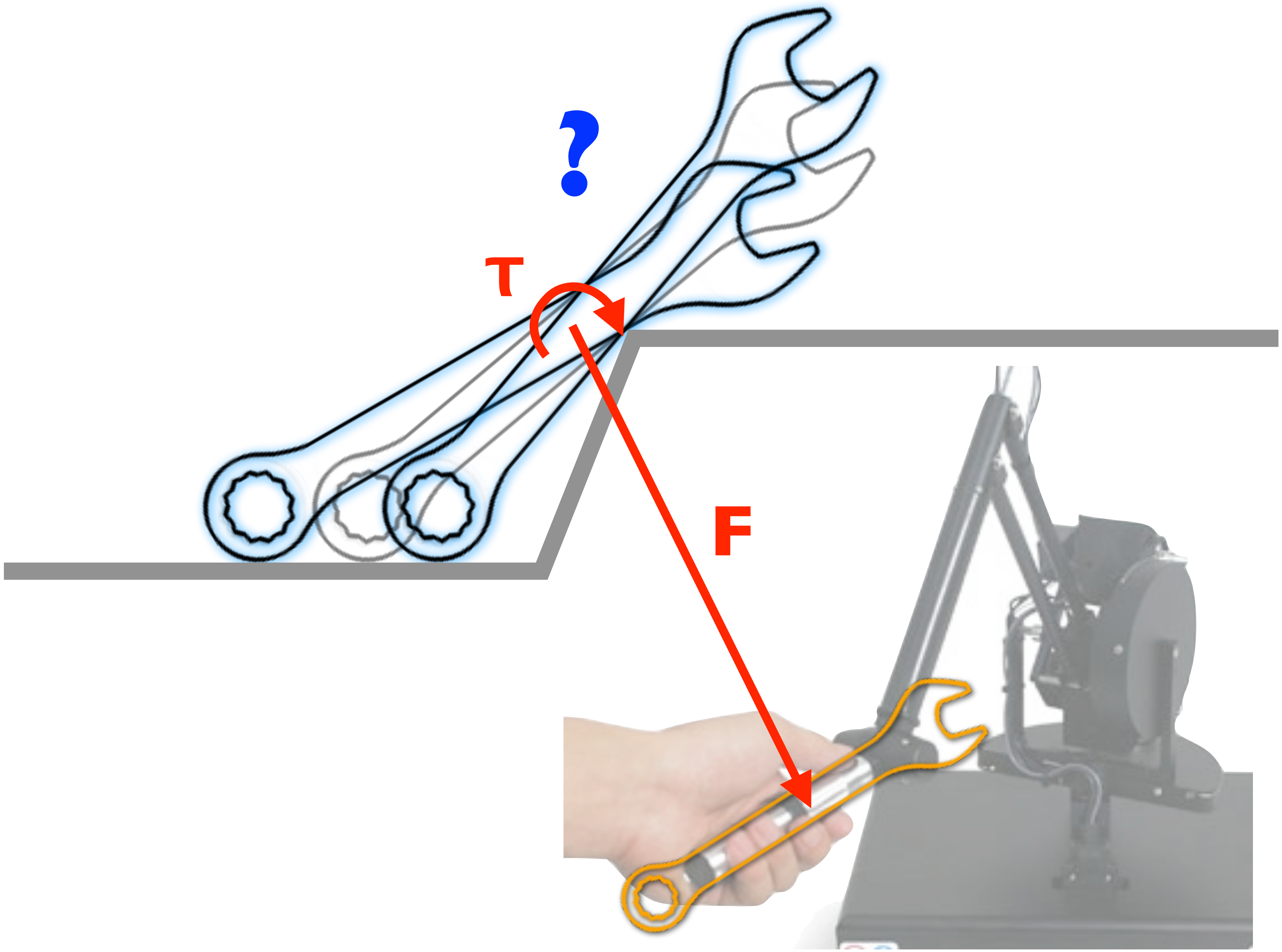
Hard Constraints



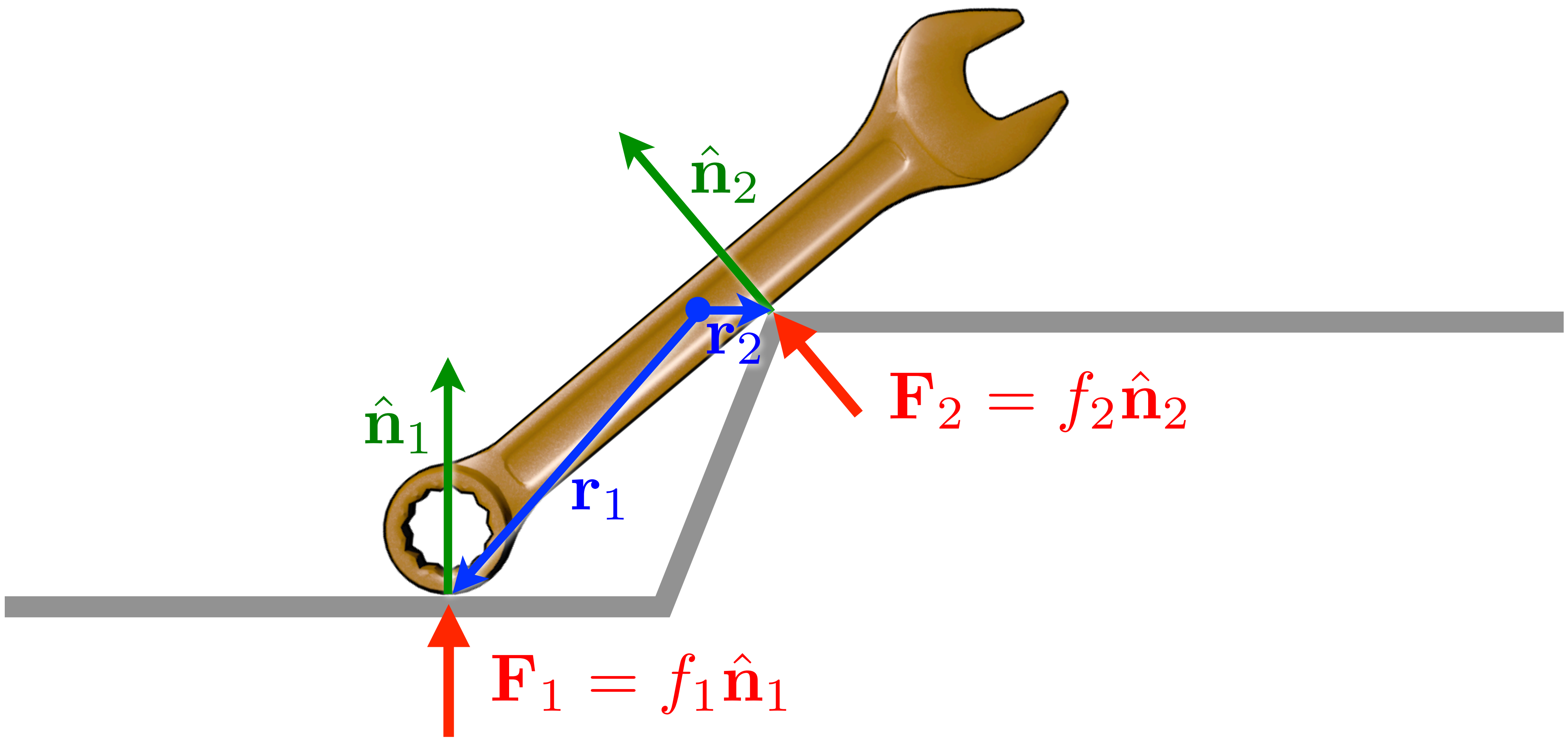
Generalized acceleration: $\mathbf{a} \equiv (\vec{a}, \vec{\alpha})$

Non-penetration constraint: $\vec{a} \cdot \hat{\mathbf{n}} + \vec{\alpha} \cdot (\mathbf{r} \times \hat{\mathbf{n}}) \geq 0$

Proxy Simulation



Solve for Contact Forces



Find f_i which satisfy: $a_i = \vec{a} \cdot \hat{\mathbf{n}}_i + \vec{\alpha} \cdot (\mathbf{r}_i \times \hat{\mathbf{n}}_i) \geq 0$

With condition: $f_i a_i = 0$

Solve for Contact Forces

- Write motion of contact points as:

$$\mathbf{a} = \mathbf{A}\mathbf{f} + \mathbf{b}$$

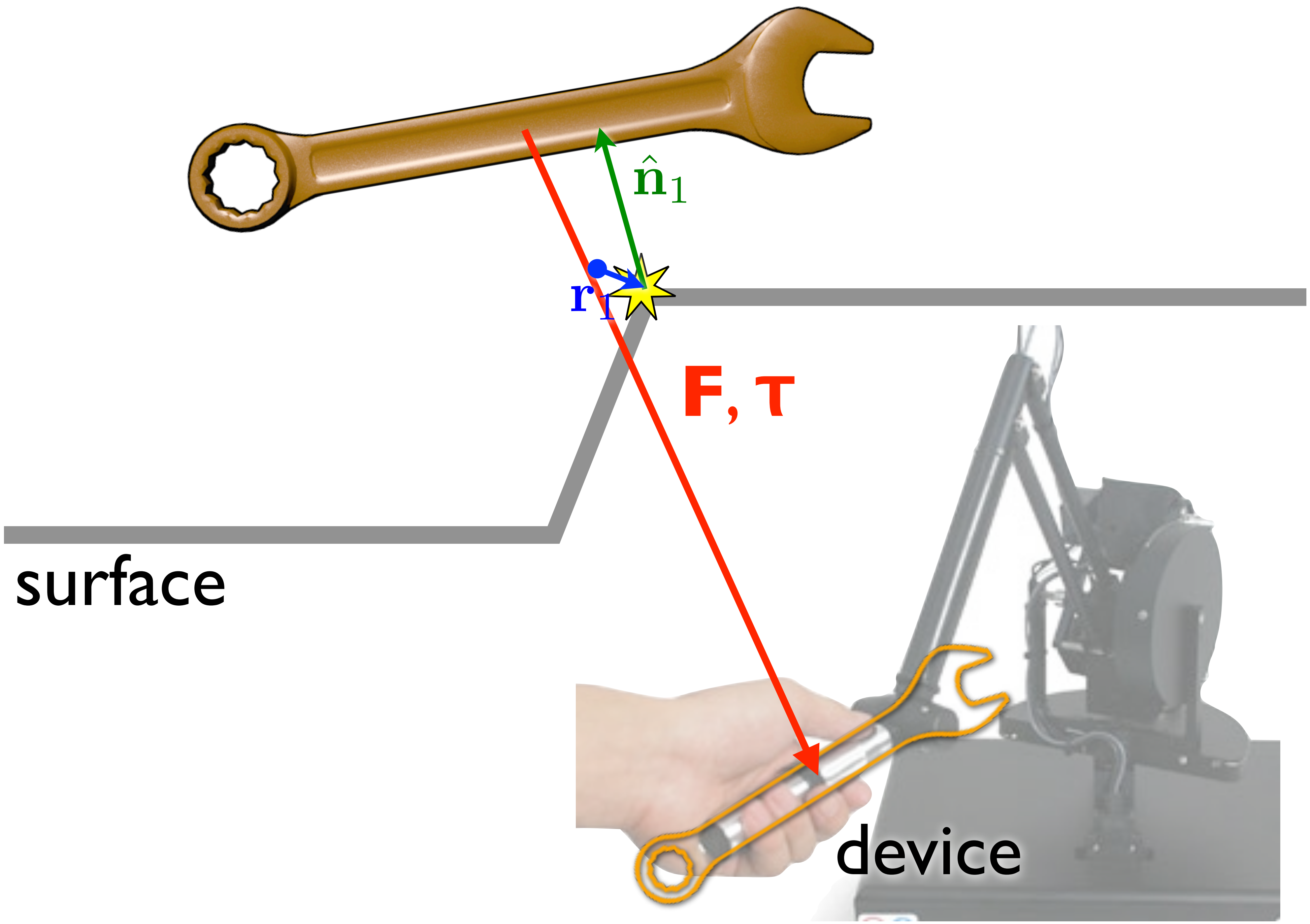
- Express conditions in matrix form:

$$\mathbf{A}\mathbf{f} + \mathbf{b} \geq \mathbf{0}, \;\; \mathbf{f} \geq \mathbf{0} \;\text{ and }\; \mathbf{f}^T(\mathbf{A}\mathbf{f} + \mathbf{b}) = \mathbf{0}$$

- Solve linear complementarity problem for **f**
- Integrate ODE to obtain position as before

[From D. Baraff, *Proc. SIGGRAPH*, 1994.]

Solve Directly for Motion



Gauss' Principle

- The proxy's *constrained* motion is that which minimizes the acceleration energy:

$$\mathbf{a}_c = \arg\min_{\mathbf{a}} \tfrac{1}{2} \left(\mathbf{F} - \mathbf{M}\mathbf{a}\right)^{\mathrm{T}} \mathbf{M}^{-1} \left(\mathbf{F} - \mathbf{M}\mathbf{a}\right)$$

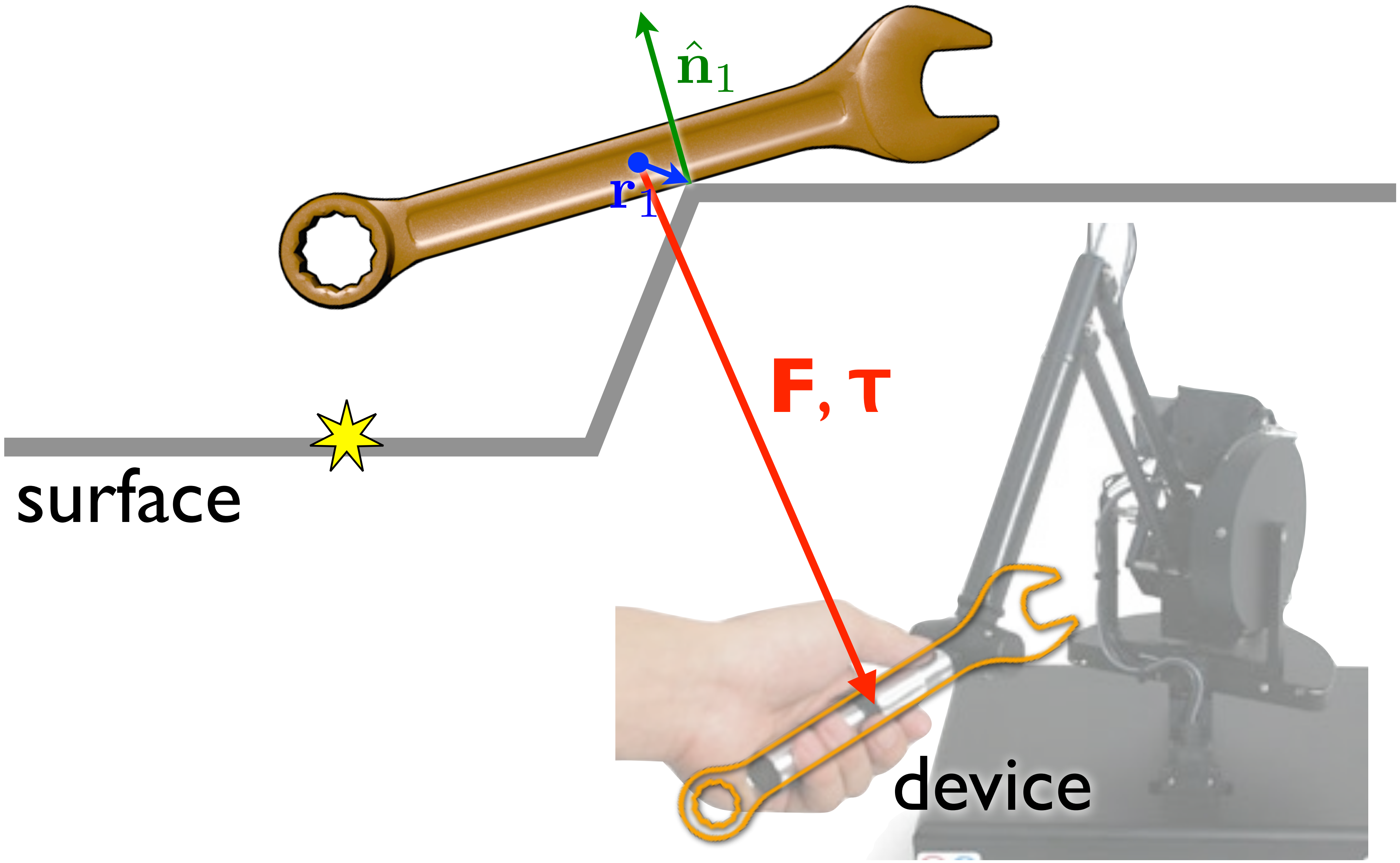
- Subject to the contact constraints:

$$\mathbf{J}_c \, \mathbf{a} \geq \mathbf{0}$$

- Solution can be obtained via quadratic programming or point projection

[From S. Redon *et al.*, *Proc. IEEE Intl. Conf. on Robotics and Automation*, 2002.]

Solve Directly for Motion

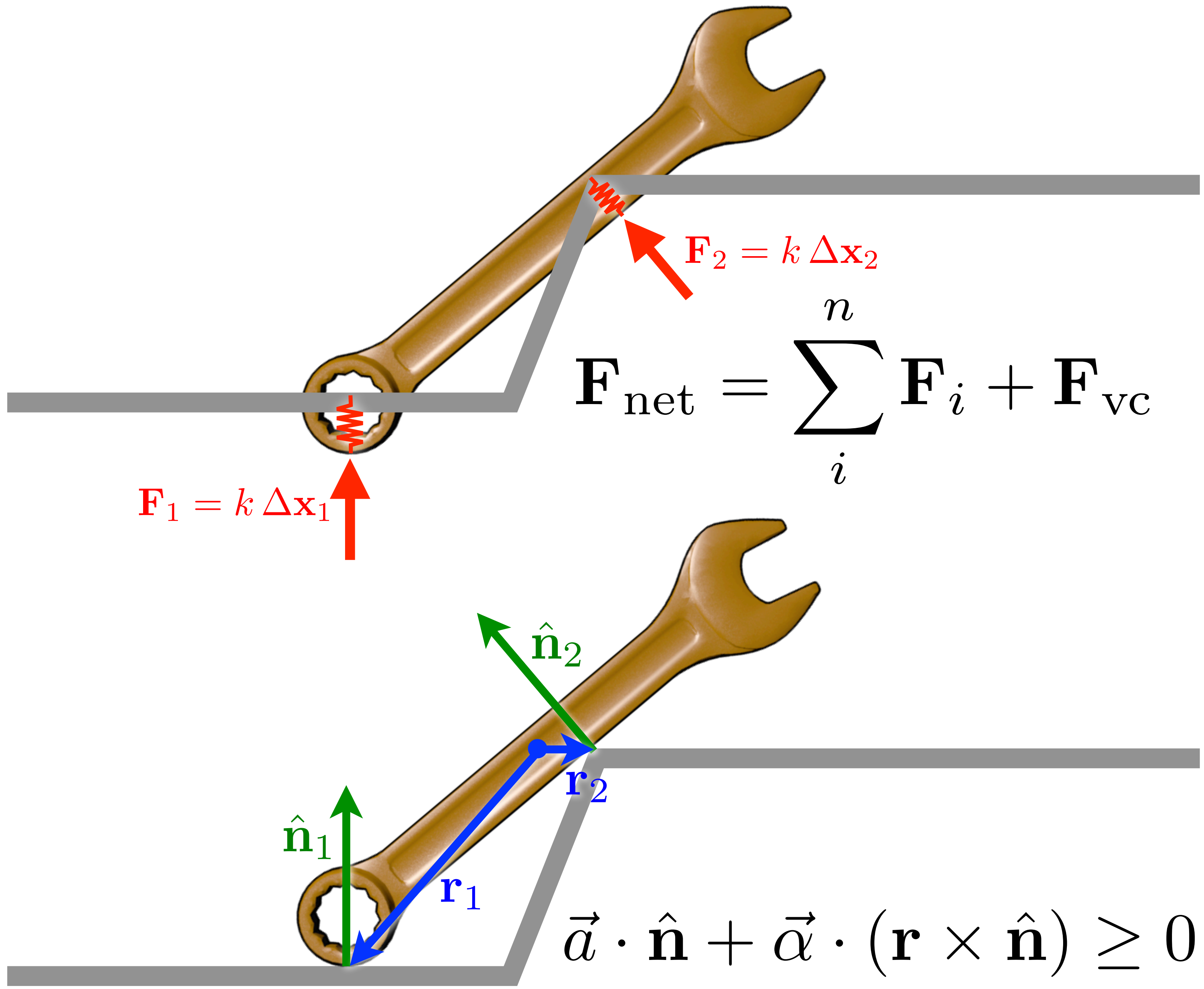


Taxonomy

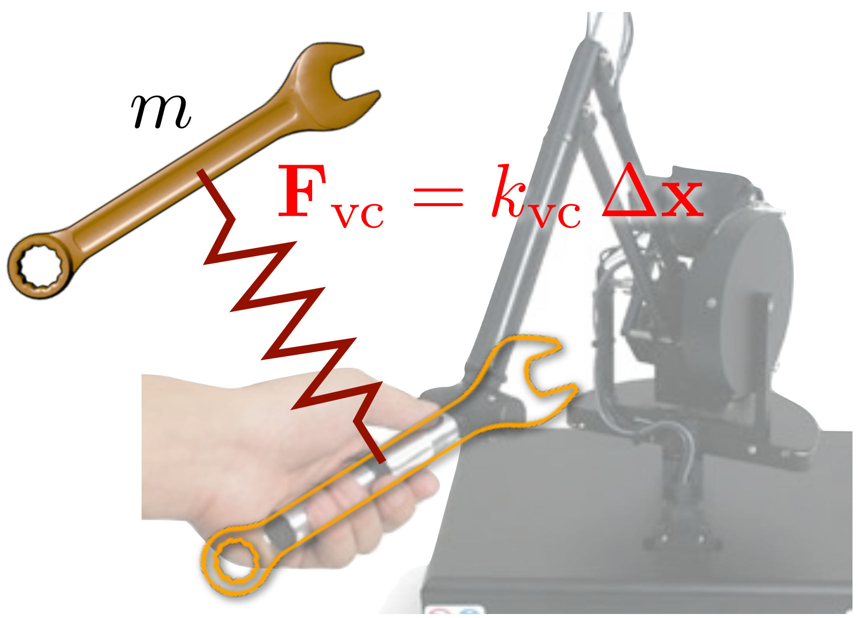
	Soft Constraints	Hard Constraints
Massless Proxy	Quasi-Static Equilibrium	Distance Minimization
Proxy with Mass	Penalty-Based Dynamics	Constrained Dynamics

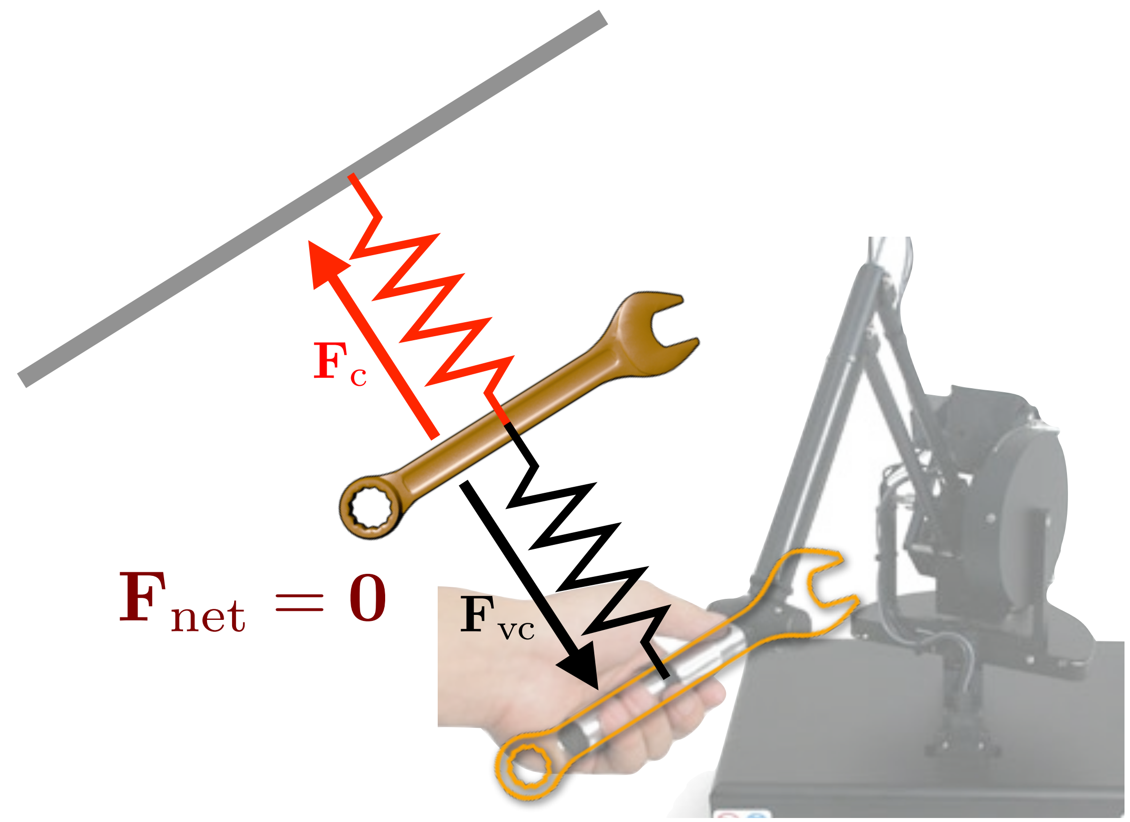
[Adapted from M. A. Otaduy *et al.*, *Proceedings of the IEEE*, 2013.]

Soft vs. Hard Constraints



Proxy With vs. Without Mass





$$m\ddot{\mathbf{x}} = \mathbf{F}_{\mathrm{net}}$$

$$\sum_i^n k\,\Delta\mathbf{x}_i + k_{\mathrm{vc}}\,\Delta\mathbf{x}_{\mathrm{vc}} = \mathbf{0}$$

Demo

Summary

- Motivation for 6-DOF haptic rendering
- Direct rendering
  - Like force fields: not very good!
- Proxy-based rendering
  - Taxonomy of proxy-based methods
- On Thursday:
  - Study examples of 6-DOF rendering methods